Reassessing D5 Product, Quotient, Chain Rules

D5: I can compute derivatives using the product, quotient, and chain rules.

Basic Preparation

1. Did you do the written practice?
2. Did you do the WeBWorK?
3. Go back to your notes, any handouts from class, the desmos activities, WeBWorK and any written practice you were able to use to prepare. Compare this to your quiz/homework.
4. Do you understand what your mistake was? If so, briefly describe what the mistake is below. If you are unsure, please go to the Calculus Center and work with a tutor until you can describe what your mistake was.

Metacognition

Now, *WHY* did you make the mistake? Answering this question is asking you to think about HOW you think about math (metacognition). Spending time here will help you become more efficient at learning math and is therefore worth the time!

1. Was your incorrect answer due to
   (a) not understanding a concept;
   (b) an error in logical reasoning (e.g., used the correct theorem/test but made the wrong conclusion, used a theorem/test/technique when it did not apply);
   (c) being careless (e.g. not reading directions, not answering the question completely, making arithmetic or basic algebra errors);
   (d) not knowing how to start or formulate an approach to the problem;
   (e) others?

   Briefly describe why your answer was incorrect:

2. What helped you recognize your mistake(s). Here are some examples: the course notes, the textbook, homework or conversations from the Calculus Center. In other words, which strategies for identifying mistakes work well for you and will help you in the future?

3. Rework the ENTIRE PROBLEM. Rewrite your solution from start to finish, carefully fixing the mistake(s) you diagnosed above. By doing the entire problem over again, you can make sure you fix your mistake and better understand the point of the exercise.

4. Describe (in detail) what you have done in order to learn from your mistake(s) and prepare for your next attempt. Did you read the textbook or class notes? Did you look at examples and/or work problems on your own or with your tutor/classmate/instructor, and if so, which problems? Did you take a different approach than listed here? (Again, the point of this isn't just to look at what you did on this problem, but how can you learn from this and be more likely to meet expectations on future assignments on the first try.)

D5: I can compute derivatives using the product, quotient, and chain rules.

- $\frac{d}{dx} f(x) \cdot g(x) = f'(x)g(x) + f(x)g'(x)$
- $\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$
- $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

Where topic was first introduced: Module 7

Videos:
Product Rule: `https://youtu.be/jMI1q8BIqa8`
Quotient Rule: `https://youtu.be/ApaBByWBv0Y`

Favorite Mistakes:

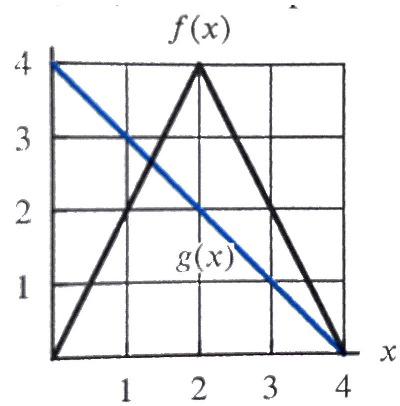
- How can you recognize when you need to use a rule? How can you parse an expression to see if you have more than one function, and if those functions are being multiplied, divided or composed? Some times (and this does NOT always work) I try to count the variables and the functions I see. For example, in $\cos(4x^{-3} \cdot e^x)$, I see three functions: cos(stuff), $4x^{-3}$ and e^x, but when I count the number of x's, I only see two. If the number of functions is more than the number of x's then a chain rule is likely needed.
- With $\frac{\ln(x^{1/2})}{\tan(x)}$, I see three functions: ln(stuff), $x^{1/2}$, and $\tan(x)$, and two x's, so there will be a chain rule.

Examples:

1. $\frac{d}{dx} 4x^{-3} \cdot e^x = -12x^{-4} \cdot e^x + 4x^{-3} \cdot e^x$

2. $\frac{d}{dx} \ln(x^{1/2}) = \frac{1}{x^{1/2}} \cdot (1/2)x^{-1/2}$

3. $\frac{d}{dx}\frac{\ln(x)}{\tan(x)} = \frac{\frac{1}{x}\tan(x) - \ln(x)\sec^2(x)}{\tan^2(x)}$



4. Use the graph to evaluate the given derivatives.
   (a) $\frac{d}{dx} f(x) \cdot g(x)|_{x=2}$ (Interpret this as take the derivative then plug in 2 for x. Then read values for $f(2)$ (y-value), $f'(2)$ (slope) etc off the graph.)

   $f'(2)g(2) + f(2)g'(2)$=DNE because $f'(2)$ DNE (corner!)

   (b) $\frac{d}{dx} \frac{f(x)}{g(x)}|_{x=1}$

   $\frac{f'(1)g(1)-f(1)g'(1)}{(g(1))^2} = \frac{2 \cdot 3 - 2 \cdot (-1)}{(3)^2}$ (using slope for derivatives and values for values)

   (c) $\frac{d}{dx} f(g(x))|_{x=3}$

   $f'(g(3)) \cdot g'(3) = f'(1) \cdot (-1) = 2(-1)$

Prepare for revision:

First, reflect on your mistake and the correct solution and what you learned: fill in the blanks "I used to think ______________but now I think ______________because I learned ______________."

In Problems **62–65**, use Figure 3.21 to evaluate the derivative.

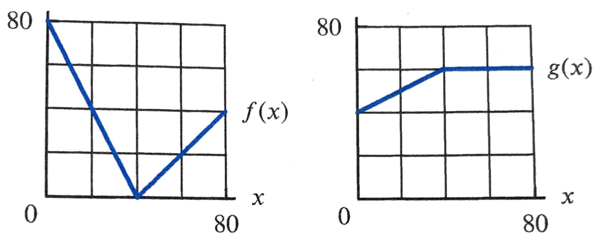


Figure 3.21

62. $\frac{d}{dx} f(g(x))|_{x=30}$

63. $\frac{d}{dx} f(g(x))|_{x=70}$

64. $\frac{d}{dx} g(f(x))|_{x=30}$

65. $\frac{d}{dx} g(f(x))|_{x=70}$

Also evaluate $\frac{d}{dx} f(x) \cdot g(x)|_{x=30}$, $\frac{d}{dx} f(x) \cdot g(x)|_{x=70}$, $\frac{d}{dx} \frac{f(x)}{g(x)}|_{x=30}$, and $\frac{d}{dx} \frac{f(x)}{g(x)}|_{x=70}$.

$$\frac{d}{dx} \sin(x)\cdot\sin(x)$$

$$\frac{d}{dx} \left(\sqrt{x}+5x^9\right)\cos x$$

$$\frac{d}{dx} \cos(x)\left(3x^2+2x-9\right)$$

$$\frac{d}{dx} \sin(x)\cos(x)$$

$$\frac{d}{dx} \left(\frac{1}{x}+\frac{1}{\sqrt[3]{x}}\right)\sin(x)$$

$$\frac{d}{dx} \left(\frac{3}{x^2}+\frac{4}{\sqrt[7]{x^3}}\right)\cos(x)$$

$$\frac{d}{dx} \frac{\cos x}{8x^3+3x} =$$

$$\frac{d}{dx} \frac{7x^4+10}{\cos x} =$$

$$\frac{d}{dx} \frac{\sin(x)+1}{\cos(x)} =$$

$$\frac{d}{dx} \frac{x^3+\sqrt{x}}{x^4} =$$

$$\frac{d}{ds} \frac{4s+3}{\sin(s)+\cos(s)} =$$

$$\frac{d}{dr} \frac{r^3+3}{\sqrt{r^3}} =$$

$$\frac{d}{dx} \cos(4x)$$

$$\frac{d}{dx} \tan(5x)$$

$$\frac{d}{dx} \sin(x^2)$$

$$\frac{d}{dx} \sin(3x^4)$$

$$\frac{d}{dx} \tan(x^4+2)$$

$$\frac{d}{dx} \cos(3x^{10})$$

More practice and solutions can be found at:
`https://www.math.colostate.edu/~freeman/CalcPrac/practice.html`