Revising D4 Basic Shortcuts

D4: I can compute derivatives for sums, constant multiples, and power, polynomial, trig, exponential, and logarithmic functions using shortcuts.

Basic Preparation

1. Did you do the written practice?
2. Did you do the WeBWorK?
3. Go back to your notes, any handouts from class, the desmos activities, WeBWorK and any written practice you were able to use to prepare. Compare this to your quiz/homework.
4. Do you understand what your mistake was? If so, briefly describe what the mistake is below. If you are unsure, please go to the Calculus Center and work with a tutor until you can describe what your mistake was.

Metacognition

Now, *WHY* did you make the mistake? Answering this question is asking you to think about HOW you think about math (metacognition). Spending time here will help you become more efficient at learning math and is therefore worth the time!

1. Was your incorrect answer due to

   (a) not understanding a concept;

   (b) an error in logical reasoning (e.g., used the correct theorem/test but made the wrong conclusion, used a theorem/test/technique when it did not apply);

   (c) being careless (e.g. not reading directions, not answering the question completely, making arithmetic or basic algebra errors);

   (d) not knowing how to start or formulate an approach to the problem;

   (e) others?

   Briefly describe why your answer was incorrect:

2. What helped you recognize your mistake(s). Here are some examples: the course notes, the textbook, homework or conversations from the Calculus Center. In other words, which strategies for identifying mistakes work well for you and will help you in the future?

3. Rework the ENTIRE PROBLEM. Rewrite your solution from start to finish, carefully fixing the mistake(s) you diagnosed above. By doing the entire problem over again, you can make sure you fix your mistake and better understand the point of the exercise.

4. Describe (in detail) what you have done in order to learn from your mistake(s) and prepare for your next attempt. Did you read the textbook or class notes? Did you look at examples and/or work problems on your own or with your tutor/classmate/instructor, and if so, which problems? Did you take a different approach than listed here? (Again, the point of this isn't just to look at what you did on this problem, but how can you learn from this and be more likely to meet expectations on future assignments on the first try.)

Where topic was first introduced: Module 7

Videos:
Power Rule: `https://youtu.be/NsovY312FKk`
Exponentials: `https://youtu.be/dh_KTrAITX8`
Trig: `https://youtu.be/U2WrUpE4ct8`
Desmos Derivative Drills

Shortcuts:

- $\frac{d}{dx}x^n = nx^{n-1}$
- $\frac{d}{dx}\sin(x) = \cos(x)$
- $\frac{d}{dx}\cos(x) = -\sin(x)$
- $\frac{d}{dx}\tan(x) = \sec^2(x)$
- $\frac{d}{dx}e^x = e^x$
- $\frac{d}{dx}a^x = a^x\ln(a)$
- $\frac{d}{dx}\ln(x) = \frac{1}{x}$
- $\frac{d}{dx}c = 0$
- $\frac{d}{dx}(f(x)+c) = \frac{d}{dx}f(x)$
- $\frac{d}{dx}c\cdot f(x) = c\frac{d}{dx}f(x)$
- $\frac{d}{dx}(f(x)+g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$

Examples:

Use the short cut rules to compute the following derivatives. Simplify algebraically first so that all of your terms are painfully obvious power functions, and then apply the power rule.

1. $f(x) = x^2 - 5x^{-1} + 12$ $f'(x) = 2x + 5x^{-2}$

2. $f(t) = \frac{2}{3t^{\frac{1}{2}}} = \frac{2}{3}t^{-1/2}$ now you can use the power rule $f'(t) = -1/3t^{-3/2}$

3. $f(x) = \sqrt{4x^3} + 6x^7 - \frac{2}{x^2} = \sqrt{4}\sqrt{x^3} + 6x^7 - 2x^{-2} = 2x^{3/2} + 6x^7 - 2x^{-2}$ and now you can take the derivative $f'(x) = 3x^{1/2} + 42x^6 + 4x^{-3}$

   Note: be very careful with notation- don't say you've taken the derivative but then write three terms, two of which you've derived and one of which you have just algebraically rewritten.

   Make sure you rewrite the terms as painfully obvious power functions (ax^p) and then use the power rule to take the derivative. While you could use the quotient or chain rules on these problems, if you are using complicated rules on simple power functions, once the derivative problems become more complex it will be taking a lot more time and brain power to use the harder rules. Please build familiarity with simplifying BEFORE deriving!

4. $g(x) = 3^x + \sin(x) + \cos(x) + \ln(x)$ $g'(x) = 3^x\ln(3) + \cos(x) - \sin(x) + \frac{1}{x}$

Favorite Mistakes

The largest "mistake" isn't really a mistake, but it's about increasing your speed and efficiency. It's also about minimizing complexity and the risk of making mistakes when the math gets much harder.

I know you can apply the quotient rule to some power functions, but when the problems become more complex, the ability to reduce the complexity will be a super power!

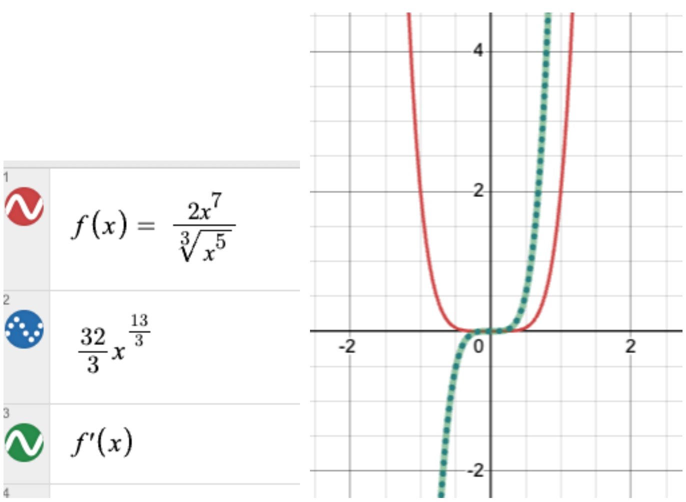
Please take the time to learn how to reframe certain fractions as painfully obvious power functions!!!

- $\frac{1}{x} = x^{-1}$. (Avoids quotient rule)
- $\sqrt[a]{x^b} = x^{b/a}$ (Avoids chain rule)
- $\frac{1}{\sqrt[a]{x^b}} = x^{-b/a}$ (Avoids quotient rule)
- $x^a \cdot x^b = x^{a+b}$ (Avoids product rule)
- $\frac{x^a}{x^b} = x^{a-b}$ (Avoids quotient rule)

Example:

$$\frac{d}{dx}\frac{2x^7}{\sqrt[3]{x^5}} = \frac{d}{dx}\frac{2x^7}{x^{5/3}} = \frac{d}{dx}2x^7 \cdot x^{-5/3} = \frac{d}{dx}2x^{7-5/3} = \frac{d}{dx}2x^{21/3-5/3} = \frac{d}{dx}2x^{16/3} = \frac{32}{3}x^{13/3}$$

(Note: you can always use Desmos to practice- make up an function like $\frac{x^c}{\sqrt[a]{x^b}}$ by choosing positive numbers for a, b and c. Enter your function in Desmos as $f(x)$, then enter your derivative, then enter $f'(x)$. If the graph of your derivative is the same as the graph of $f'(x)$, you got it! If not, start trouble shooting your algebra- you can ask desmos to graph your intermediate steps and see if the graph still looks like $f(x)$. If it doesn't you probably made an algebra mistake somewhere.)



Prepare for revision:

First, reflect on your mistake and the correct solution and what you learned: fill in the blanks “I used to think but now I think ________________ because I learned ________________.”

- $\dfrac{d}{dx}3x^4$
- $\dfrac{d}{dx}4\sin(x) =$
- $\dfrac{d}{dx}7\cos(x) =$
- $\dfrac{d}{dx}(-2\tan(x)) =$
- $\dfrac{d}{dx}e^x =$
- $\dfrac{d}{dx}6^x =$
- $\dfrac{d}{dx}(\ln(x) + \ln(2)) =$

  (do not use quotient rule on the following problems:)

- $\dfrac{d}{dx}3x^{-4}$
- $\dfrac{d}{dx}\dfrac{3x}{x^4}$
- $\dfrac{d}{dx}\dfrac{3x}{x^{1/4}}$
- $\dfrac{d}{dx}\left(\dfrac{3}{x} + \dfrac{x}{3}\right)$