Revising D3 Graphically

D3: I can use information about f, f', and/or f" to identify and/or draw accurate graphs of the other functions.

Basic Preparation

1. Did you do the written practice?
2. Did you do the WeBWorK?
3. Go back to your notes, any handouts from class, the desmos activities, WeBWorK and any written practice you were able to use to prepare. Compare this to your quiz/homework.
4. Do you understand what your mistake was? If so, briefly describe what the mistake is below. If you are unsure, please go to the Calculus Center and work with a tutor until you can describe what your mistake was.

Metacognition

Now, *WHY* did you make the mistake? Answering this question is asking you to think about HOW you think about math (metacognition). Spending time here will help you become more efficient at learning math and is therefore worth the time!

1. Was your incorrect answer due to
   (a) not understanding a concept;
   (b) an error in logical reasoning (e.g., used the correct theorem/test but made the wrong conclusion, used a theorem/test/technique when it did not apply);
   (c) being careless (e.g. not reading directions, not answering the question completely, making arithmetic or basic algebra errors);
   (d) not knowing how to start or formulate an approach to the problem;
   (e) others?

   Briefly describe why your answer was incorrect:

2. What helped you recognize your mistake(s). Here are some examples: the course notes, the textbook, homework or conversations from the Calculus Center. In other words, which strategies for identifying mistakes work well for you and will help you in the future?

3. Rework the ENTIRE PROBLEM. Rewrite your solution from start to finish, carefully fixing the mistake(s) you diagnosed above. By doing the entire problem over again, you can make sure you fix your mistake and better understand the point of the exercise.

4. Describe (in detail) what you have done in order to learn from your mistake(s) and prepare for your next attempt. Did you read the textbook or class notes? Did you look at examples and/or work problems on your own or with your tutor/classmate/instructor, and if so, which problems? Did you take a different approach than listed here? (Again, the point of this isn't just to look at what you did on this problem, but how can you learn from this and be more likely to meet expectations on future assignments on the first try.)

Where topic was first introduced: Module 5

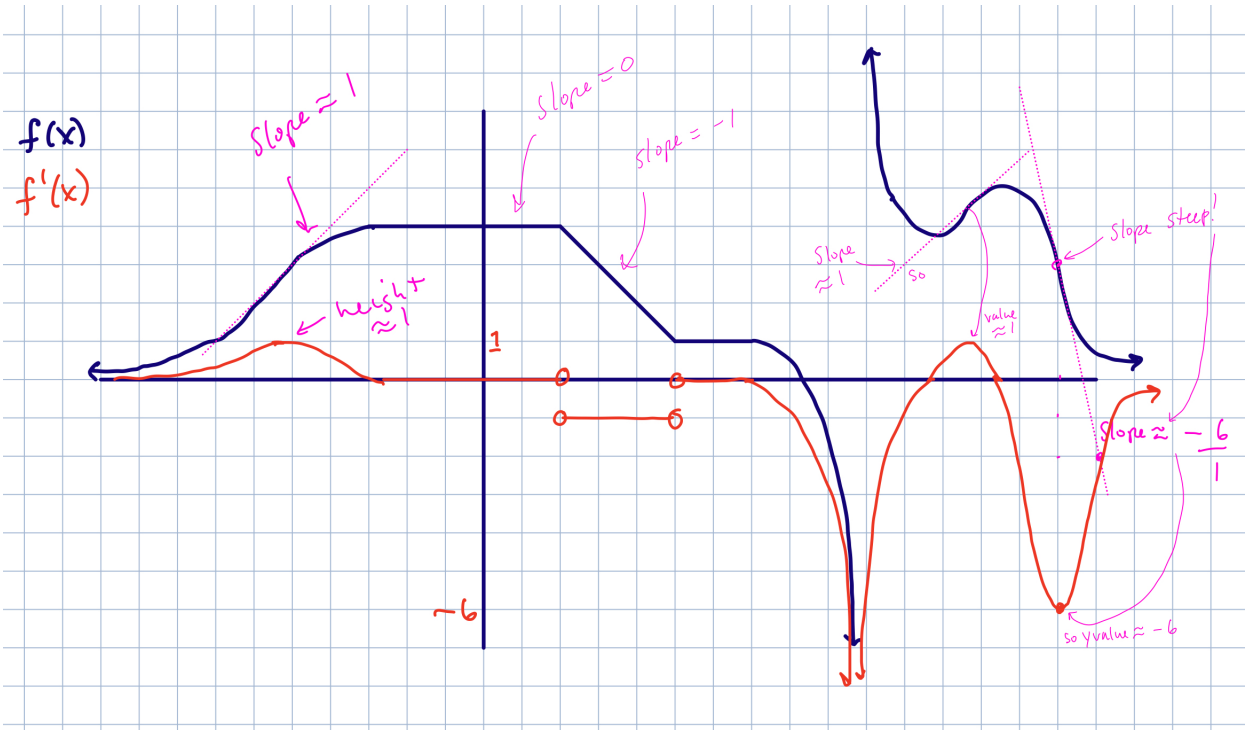
Video showing what f' can tell us about f: `https://youtu.be/IpxR9qFeA0s`

Favorite Mistakes:

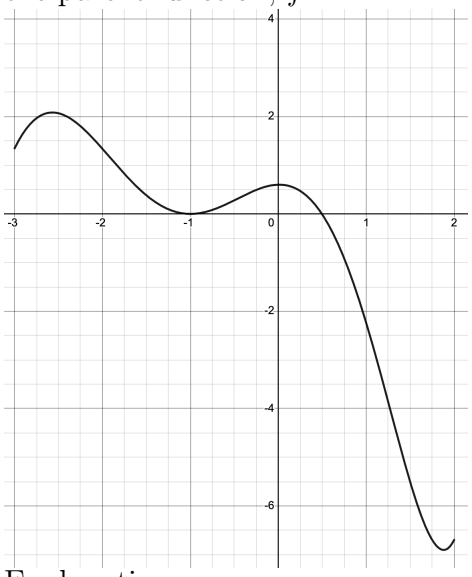
- Not realizing the graph is of the derivative already.
- Not interpreting f' as slope, and then drawing a function f with that slope.
- Not using increasing or decreasing of the function to know to sketch the graph of the derivative as above or below the x-axis.
- If $f'(a) = 0$ then then f is neither increasing nor decreasing when $x = a$ and should have a horizontal slope at that point, even if the derivative is positive on either side (meaning the parent function is increasing on either side.)

Example:

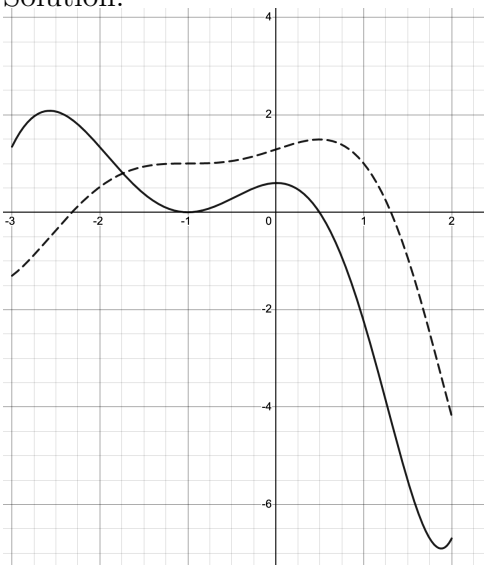
1. Sketch the graph of a function such that:
   - If $f'(x) > 0$ then $f(x)$ is increasing.
   - If $f'(x) < 0$ then $f(x)$ is decreasing.
   - If $f'(x) = 0$ then $f(x)$ is constant or neither increasing nor decreasing, or possibly changing direction.
   - The steepness of the slope is related to the magnitude of the derivative graph. In other words, if f has a steep slope, then the values of f' will be far away from the x-axis.



2. Interpret the graph below as being the derivative. Sketch a possible graph of the parent function, f.

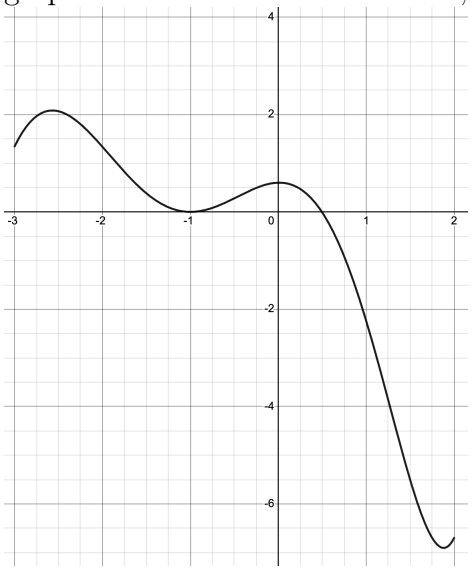


Solution:

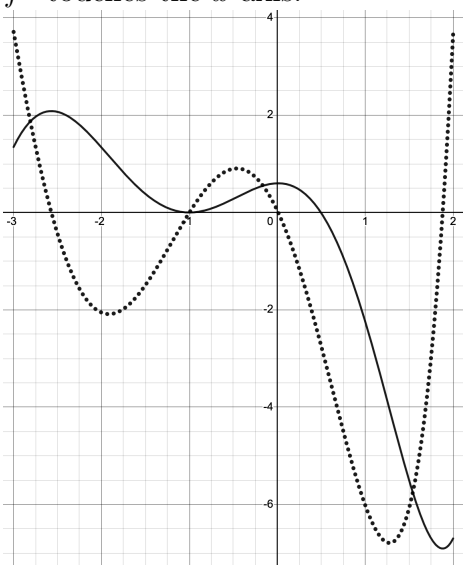


Explanation:
Interpret the derivative graph as telling us the slopes of the parent function. Because f' is positive on $(-3,-1)$ and $(-1,0.5)$, the parent function should be increasing on those intervals. Pay attention to $x=-1$ and make sure the parent graph has a horizontal tangent slope there. Because f' is 0 at -1, and 0.5, the parent function has a horizontal tangent line and possible direction change. Also pay attention to the magnitude of the graph- for example $f'(1.5)$ has a large $|y|$ value, so the parent function should be steep at those x values. Note that the parent function can be shifted vertically so there are many possible solutions.

3. Interpret the graph below as being the derivative. Sketch a possible graph of the second derivative, f''.



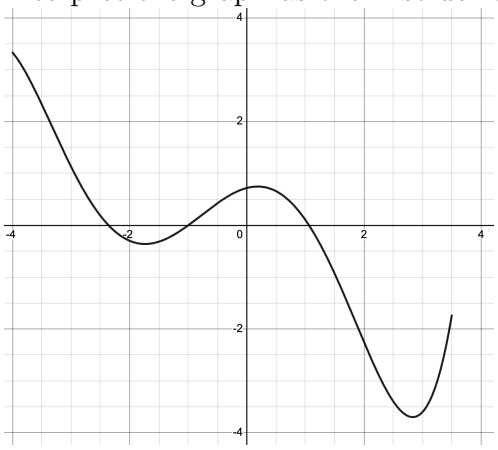
Solution: If f' is increasing, then f'' needs to be above the x axis. Where f' has a direction change, f'' touches the x axis.



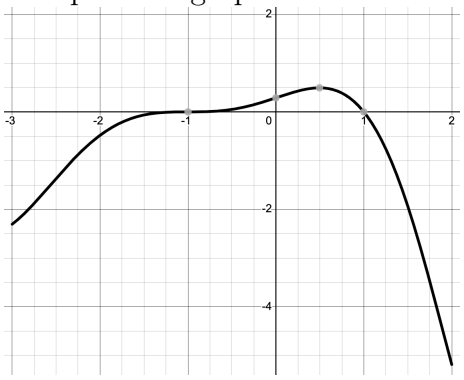
Prepare for revision:

First, reflect on your mistake and the correct solution and what you learned: fill in the blanks "I used to think _____but now I think _____because I learned _____."

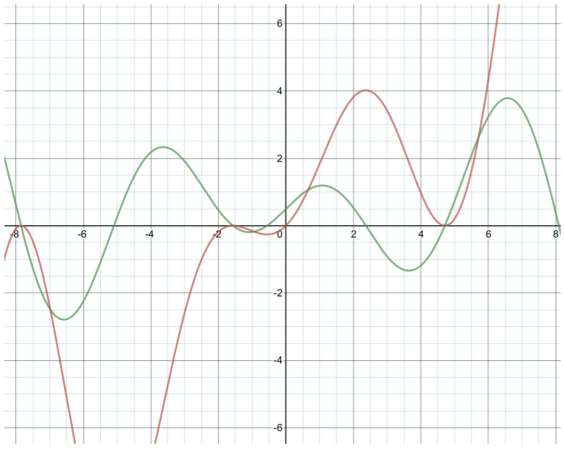
1. Interpret the graph as the first derivative. Sketch the parent function and the second derivative.



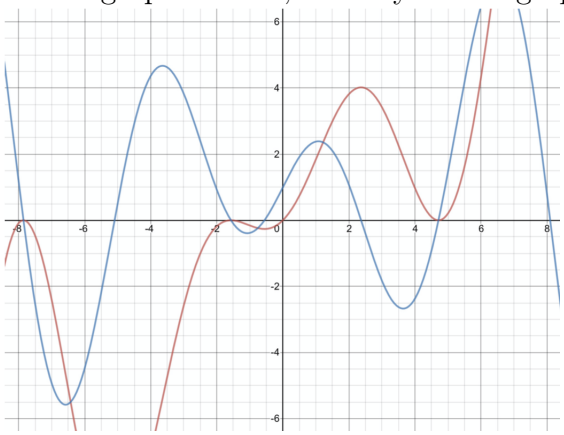
2. Interpret the graph as the first derivative. Sketch the parent function and the second derivative.



3. In the graphs below, identify which graph is f and which is f'. Pay attention to magnitude!



4. In the graphs below, identify which graph is f and which is f'. Pay attention to magnitude!



5. The bold graph is $f(x)$. Which of the other two graphs is the derivative?