Revising D3 - Algebraically

D3: I can use the first and second derivatives to describe the behavior of functions, including listing intervals of increasing/decreasing, concavity, extreme values.

Basic Preparation

1. Did you do the written practice?
2. Did you do the WeBWorK?
3. Go back to your notes, any handouts from class, the desmos activities, WeBWorK and any written practice you were able to use to prepare. Compare this to your quiz/homework.
4. Do you understand what your mistake was? If so, briefly describe what the mistake is below. If you are unsure, please go to the Calculus Center and work with a tutor until you can describe what your mistake was.

Metacognition

Now, *WHY* did you make the mistake? Answering this question is asking you to think about HOW you think about math (metacognition). Spending time here will help you become more efficient at learning math and is therefore worth the time!

1. Was your incorrect answer due to
   (a) not understanding a concept;
   (b) an error in logical reasoning (e.g., used the correct theorem/test but made the wrong conclusion, used a theorem/test/technique when it did not apply);
   (c) being careless (e.g. not reading directions, not answering the question completely, making arithmetic or basic algebra errors);
   (d) not knowing how to start or formulate an approach to the problem;
   (e) others?

   Briefly describe why your answer was incorrect:

2. What helped you recognize your mistake(s). Here are some examples: the course notes, the textbook, homework or conversations from the Calculus Center. In other words, which strategies for identifying mistakes work well for you and will help you in the future?

3. Rework the ENTIRE PROBLEM. Rewrite your solution from start to finish, carefully fixing the mistake(s) you diagnosed above. By doing the entire problem over again, you can make sure you fix your mistake and better understand the point of the exercise.

4. Describe (in detail) what you have done in order to learn from your mistake(s) and prepare for your next attempt. Did you read the textbook or class notes? Did you look at examples and/or work problems on your own or with your tutor/classmate/instructor, and if so, which problems? Did you take a different approach than listed here? (Again, the point of this isn't just to look at what you did on this problem, but how can you learn from this and be more likely to meet expectations on future assignments on the first try.)

D3: I can use the first and second derivatives to describe the behavior of functions, including listing intervals of increasing/decreasing, concavity, extreme values.

- If $f' > 0$, then f is increasing. If $f' < 0$, then f is decreasing.
- If $f'' > 0$, then f' is increasing, and f is concave up.
- If $f'' < 0$, then f' is decreasing, and f is concave down.
- Start by finding algebraically where f' is zero or undefined, then test intervals for increase/decrease. For concavity, do the same but look at where f'' is zero or undefined.

Where topic was first introduced: Modules 4, 5, and 10

Videos:

- Video on information about the derivative from the original function
`https://www.youtube.com/watch?v=SYyecwhLBm8&feature=youtu.be`
- Video on information about the original function from the derivative
`https://youtu.be/IpxR9qFeA0s`

Favorite Mistakes:

- Remember to address when/if f' undefined- this is a place where f could have a direction change or even a critical point.
- f can't have a critical point at c if c is not in the domain of f. (In other words, check to see if $f(c)$ is defined.
- If $f(a) = 0$ then f cannot be increasing when $x = a$. This impacts how you list your intervals of increase/decrease. (Don't include a in your intervals!)
- Name maximum/minimum values with output numbers. Or, say f has a max/min when $x = a$. (using input numbers)
- If technology shows you a different graph with different extrema than you found, trace back through your work and try to figure out why there is a discrepancy, and if it's from your work or from the technology.

Example:

List all intervals of increase/decrease, critical points, intervals of concavity, inflection points and extrema.
$f(x) = xe^{-x^2}$

1. Increase/decrease/critical points:

$$f'(x) = e^{-x^2} - 2x^2e^{-x^2} = \frac{1}{e^{x^2}} - \frac{2x^2}{e^{x^2}} = \frac{1 - 2x^2}{e^{x^2}}.$$

f' is always defined because e^{x^2} is never 0. $f' = 0$ if $1 - 2x^2 = 0$ or if $x = \pm\sqrt{1/2}$.

$f'(-10) < 0$, $f'(0) > 0$ and $f'(10) < 0$ tells us that f is increasing for $(-\sqrt{1/2}, \sqrt{1/2})$ and f is decreasing on $(-\infty, -\sqrt{1/2})$ and $(\sqrt{1/2}, \infty)$.

f has critical points at $x = -\sqrt{1/2}$ and $\sqrt{1/2}$. (Double check domain: f is continuous for all x because it's a product of continuous functions, so both points are in the domain of f.)

2. Concavity/Inflection Points

   $f''(x) = \dfrac{-4xe^{x^2} - (2x-4x^3)e^{x^2}}{(e^{x^2})^2} = \dfrac{e^{x^2}(-4x-2x+4x^3)}{(e^{x^2})^2} = \dfrac{-6x+4x^3}{e^{x^2}} = \dfrac{x(-6+4x^2)}{e^{x^2}}$

   $f''(x) = 0$ if $x = 0$ or $x = \pm\sqrt{3/2}$. $f''(x)$ is never undefined because e^{x^2} is never zero.

   $f''(-10) < 0$, $f''(-\sqrt{1/2}) > 0$, $f''(\sqrt{1/2}) < 0$ and $f''(10) > 0$ Thus f is concave up on $(-\sqrt{3/2}, 0)$ and $(\sqrt{3/2}, \infty)$ and f is concave down on $(-\infty, -\sqrt{3/2})$ and $0, \sqrt{3/2})$
   Because the concavity changes, we have inflection points when $x = -\sqrt{3/2}$, 0, $\sqrt{3/2}$.

3. Testing For Extrema at Critical Points using First Derivative Test:
   Because f' changes from negative to positive, f has a minimum when $x = -\sqrt{1/2}$. Because f' changes from positive to negative, f has a maximum when $x = \sqrt{1/2}$.

4. Using the Second Derivative Test instead:
   $f''(-\sqrt{1/2}) > 0$ implies a local minimum, and $f''(\sqrt{1/2}) < 0$ implies a local maximum.

Prepare for revision:

First, reflect on your mistake and the correct solution and what you learned: fill in the blanks "I used to think ________________but now I think ________________because I learned ________________."

For each function below, list all intervals of increase/decrease, critical points, intervals of concavity, inflection points and extrema. Use calculus to justify all of your work, and use technology to check your work.

- $f(x) = x^3 - 3x^2 + 20$ on $-1 \le x \le 3$. (Then do this for $(-\infty, \infty)$ also)
- $f(x) = x^4 - 8x^2$ on $-3 \le x \le 1$
- $f(x) = 3x^{1/3} - x$ on $-1 \le x \le 8$
- $f(x) = x^2 - 2\ln(x)$ on $-3 \le x \le 4$
- $f(x) = \dfrac{x+1}{x^2+3}$ on $-1 \le x \le 2$

Pick a derivative then answer the questions:

1. $f'(x) = 5x^2$
2. $g'(x) = -2x + 10$
3. $h'(x) = \sin(x)$
4. $j'(x) = e^x$

Questions:

- List an x value where the parent function (f, g, h or j) is A) increasing B) decreasing C) Neither or explain why this is not possible. How do you know?
- Compute the second derivative.
- List an x value where the parent function (f, g, h or j) is A) concave up B) concave down C) Neither or explain why this is not possible. How do you know?