Revising D2

D2: I can find the equation of the line tangent to a function at a point and use this line as a linear approximation to estimate the value of a function at nearby points.

Basic Preparation

1. Did you do the written practice?
2. Did you do the WeBWorK?
3. Go back to your notes, any handouts from class, the desmos activities, WeBWorK and any written practice you were able to use to prepare. Compare this to your quiz/homework.
4. Do you understand what your mistake was? If so, briefly describe what the mistake is below. If you are unsure, please go to the Calculus Center and work with a tutor until you can describe what your mistake was.

Metacognition

Now, *WHY* did you make the mistake? Answering this question is asking you to think about HOW you think about math (metacognition). Spending time here will help you become more efficient at learning math and is therefore worth the time!

1. Was your incorrect answer due to
   (a) not understanding a concept;
   (b) an error in logical reasoning (e.g., used the correct theorem/test but made the wrong conclusion, used a theorem/test/technique when it did not apply);
   (c) being careless (e.g. not reading directions, not answering the question completely, making arithmetic or basic algebra errors);
   (d) not knowing how to start or formulate an approach to the problem;
   (e) others?

   Briefly describe why your answer was incorrect:

2. What helped you recognize your mistake(s). Here are some examples: the course notes, the textbook, homework or conversations from the Calculus Center. In other words, which strategies for identifying mistakes work well for you and will help you in the future?

3. Rework the ENTIRE PROBLEM. Rewrite your solution from start to finish, carefully fixing the mistake(s) you diagnosed above. By doing the entire problem over again, you can make sure you fix your mistake and better understand the point of the exercise.

4. Describe (in detail) what you have done in order to learn from your mistake(s) and prepare for your next attempt. Did you read the textbook or class notes? Did you look at examples and/or work problems on your own or with your tutor/classmate/instructor, and if so, which problems? Did you take a different approach than listed here? (Again, the point of this isn't just to look at what you did on this problem, but how can you learn from this and be more likely to meet expectations on future assignments on the first try.)

Where topic was first introduced: Module 6

Prerequisite Knowledge:

Writing the equation of a line requires either two points or one point and a slope. Tangent lines are the basis of understanding derivatives, so we usually have the situation of one point and a slope for writing a tangent line equation.

Equations of lines:

- $y = mx + b$ (slope intercept form- usually requires more work because you have to find b and the plug back in)
- $y = m(x - x_0) + y_0$ (point slope form- easiest form for calculus)
- $y = f'(x_0)(x - x_0) + f(x_0)$ (point slope form but with calculus notation)

Note that concavity tells us if we have an overestimate or underestimate. A function that is concave up has tangent lines below the function, so a tangent line approximation will be an underestimate.

Favorite Mistakes:

- The linearization isn't actually a line.
- Not using concavity to determine over/under estimate.

Examples:

1. Write the equation of a line tangent to $g(x)$ at $x = 2$ given that $g(2) = 5$ and $g'(2) = -10$.

   $y = -10(x - 2) + 5$

2. Write the equation of the line tangent to $h(x) = \ln(x)$ at $x = 1$.

   $h'(x) = 1/x$, and $h'(1) = 1/1 = 1$. $h(1) = \ln(1) = 0$. Thus the tangent line is $y = 1(x - 1) + 0$ or $y = x - 1$.

3. Use a tangent line to approximate $\ln(1.001)$.

   We know $\ln(1) = 0$, $\frac{d}{dx}\ln(x)|_{x=1} = \frac{1}{x}|_{x=1} = 1$, and $x = 1$ is close to $x = 1.001$. So, if x is close to 1, $\ln(x)$ is close to $y = x - 1$. Thus $\ln(1.001) \approx 1.001 - 1 = 0.001$.

4. Let $f(2) = 78$ and $f'(2) = -123$. Write the equation of the line tangent to $f(x)$ when $x = 2$ and use it to approximate $f(2.04)$.

   The x value is 2, the corresponding output ($f(2)$) is 78 and the slope ($f'(2)$) is -123. Thus the tangent line is $y = -123(x - 2) + 78$. If you want, you can simplify, but unless directions say to do so, it isn't needed. Finally, $f(2.04) \approx -123(2.04 - 2) + 78 = 73.08$

5. Let $xy + y^2 = 15$. Write the equation of a line tangent to this curve at the point $(2, 3)$.

   $\frac{d}{dx}(xy + y^2 = 15) \implies y + xy' + 2yy' = 0 \implies xy' + 2yy' = -y \implies y'(x + 2y) = -y$
   $\implies y' = \frac{-y}{x + 2y}$
   At $(2, 3)$ $y' = \frac{-3}{2 + 2 \cdot 3} = -\frac{3}{8}$
   Tangent Line: $\hat{y} = -\frac{3}{8}(x - 2) + 3$

   Use this tangent line to find an approximation for y if $x = 2.01$.

   $\hat{y}(2.01) = -\frac{3}{8}(2.01 - 2) + 3 = -0.375(0.01) + 3 = -0.00375 + 3 = 2.99625$

Prepare for revision:

First, reflect on your mistake and the correct solution and what you learned: fill in the blanks "I used to think _____but now I think _____because I learned _____."

1. Let $f(6) = 456$ and $f'(6) = 653$. Write the equation of the line tangent to $f(x)$ when $x = 6$ and approximate $f(5.9)$. If $f''(6) < 0$ do you have an underestimate or an overestimate?

2. Let $f(8) = 7$ and $f'(8) = -90$. Write the equation of the line tangent to $f(x)$ when $x = 8$ and approximate $f(7.9)$. If $f''(8) > 0$ do you have an underestimate or an overestimate?

3. Let $f(10) = 8$ and $f'(10) = 63$. Write the equation of the line tangent to $f(x)$ when $x = 10$ and approximate $f(9.9)$.

4. Let $f(12) = -68$ and $f'(12) = -13$. Write the equation of the line tangent to $f(x)$ when $x = 12$ and approximate $f(11.9)$.

5. Let $f(32) = -90$ and $f'(32) = 34$. Write the equation of the line tangent to $f(x)$ when $x = 32$ and approximate $f(31.9)$.

6. Let $-xy + x^2 = 8$. Write the equation of a line tangent to this curve at the point $(4, 2)$, and use the tangent line to approximate y if $x = 4.01$.

7. Let $y^3 - x^3y^2 = -1$. Write the equation of a line tangent to this curve at the point $(2, 1)$, and use the tangent line to approximate y if $x = 2.01$.