Reassessing L3

L3: I can determine the points at which a function is (and is not) continuous both graphically and algebraically.

Basic Preparation

1. Did you do the written practice?
2. Did you do the WeBWorK?
3. Go back to your notes, any handouts from class, the desmos activities, WeBWorK and any written practice you were able to use to prepare. Compare this to your quiz/homework.
4. Do you understand what your mistake was? If so, briefly describe what the mistake is below. If you are unsure, please go to the Calculus Center and work with a tutor until you can describe what your mistake was.

Metacognition

Now, *WHY* did you make the mistake? Answering this question is asking you to think about HOW you think about math (metacognition). Spending time here will help you become more efficient at learning math and is therefore worth the time!

1. Was your incorrect answer due to
   (a) not understanding a concept;
   (b) an error in logical reasoning (e.g., used the correct theorem/test but made the wrong conclusion, used a theorem/test/technique when it did not apply);
   (c) being careless (e.g. not reading directions, not answering the question completely, making arithmetic or basic algebra errors);
   (d) not knowing how to start or formulate an approach to the problem;
   (e) others?

   Briefly describe why your answer was incorrect:

2. What helped you recognize your mistake(s). Here are some examples: the course notes, the textbook, homework or conversations from the Calculus Center. In other words, which strategies for identifying mistakes work well for you and will help you in the future?

3. Rework the ENTIRE PROBLEM. Rewrite your solution from start to finish, carefully fixing the mistake(s) you diagnosed above. By doing the entire problem over again, you can make sure you fix your mistake and better understand the point of the exercise.

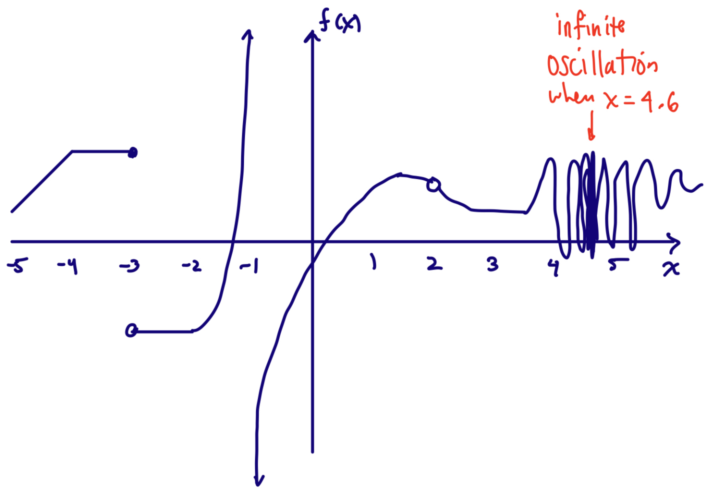
4. Describe (in detail) what you have done in order to learn from your mistake(s) and prepare for your next attempt. Did you read the textbook or class notes? Did you look at examples and/or work problems on your own or with your tutor/classmate/instructor, and if so, which problems? Did you take a different approach than listed here? (Again, the point of this isn't just to look at what you did on this problem, but how can you learn from this and be more likely to meet expectations on future assignments on the first try.)

Where topic was first introduced: Module 2

Video: `https://youtu.be/YhF01FGJRXU`

Example:

1. List all places where $f(x)$ is not continuous. List two x values where $f(x)$ IS continuous.



- $f(x)$ has a jump discontinuity when $x = -3$.
- $f(x)$ has an infinite discontinuity when $x = -1$.
- $f(x)$ has a removeable (hole) discontinuity when $x = 2$. Favorite Mistake!
- $f(x)$ has an infinite oscillation discontinuity when $x = 4.6$.
- $f(x)$ is continuous when $x = 0$ and $x = 3$.

2. Find all values of x where $h(x)$ discontinuous or explain why $h(x)$ is always continuous if $h(x) = \begin{cases} 3x+7 & x \geq 2 \\ 3x+6 & x < 2 \end{cases}$.

   $3x + 7$ and $3x + 6$ are continuous for all x (because they are linear functions). We need to check when we change from one piece to the next, so at $x = 2$. We check continuity here with limits:

   $\lim_{x \to 2^+} h(x) = \lim_{x \to 2^+} (3x + 7) = 3(2) + 7 = 13.$

   $\lim_{x \to 2^-} h(x) = \lim_{x \to 2^-} (3x + 6) = 3(2) + 6 = 12.$

   $13 \neq 12$ therefore $\lim_{x \to 2} h(x)$ does not exist and h fails to be continuous when $x = 2$.

   Consequences of continuity: if a function is not continuous at a points $x = a$ then the function cannot be differentiable at that point.

   Also, be sure to check for division by 0 on each piece, then cross-reference that with the domain of each piece.

3. Find all values of x where $f(x)$ discontinuous or explain why $f(x)$ is always continuous. Reference the bullet from the definition of continuity to justify your answers.

$$f(x) = \begin{cases} \frac{x}{(x-5)(x+6)} & x \le 0 \\ \frac{1}{\cos(x)} & 0 < x < \pi \\ \frac{42}{x(x-4)} & x > \pi \end{cases}$$

First, check each piece for division by zero. (Set the denominators =0).

- $\frac{x}{(x-5)(x+6)}$ is undefined when $x = 5$ and $x = -6$.
- $\cos(x)$ is undefined when $x = \pi/2$
- $\frac{42}{x(x-4)}$ is undefined when $x = 0$ and $x = 4$

Now list the values for x where the function changes definition: $x = 0$ and $x = \pi$

Finally, check each x value with the definition to verify and identify which part of the definition of continuity applies:

$x = 5$ Since $5 > \pi$, $f(5) = \dfrac{42}{5(5-4)}$, which exists, and that piece of the function IS continuous at all values other than 0 and 4, so we eliminate $x = 5$ from our list.

$x = -6$ Since $-6 < 0$ and $f(-6)$ is undefined (division by 0), f is discontinuous at $x = -6$.

$x = \pi/2$ Since $0 < \pi/2 < \pi$, and $f(\pi/2) = \frac{42}{\cos(\pi/2)}$ is undefined because $\cos(\pi/2) = 0$, f is discontinuous at $x = \pi/2$.

$x = 0$ $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} \frac{x}{(x-5)(x+6)} = \frac{0}{(0-5)(0+6)} = \frac{0}{(-5)(6)} = 0$

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} \frac{1}{\cos(x)} = \frac{1}{\cos(0)} = \frac{1}{1} = 1$. Since $0 \neq 1$, the limit does not exist, and f is discontinuous at $x = 0$.

$x = 4$ Since $4 > \pi$, $f(4) = \frac{42}{4(4-4)}$ which is undefined so f is discontinuous at $x = 4$.

$x = \pi$ $\lim\limits_{x\to \pi^-} f(x) = \lim\limits_{x\to \pi^-} \frac{1}{\cos(x)} = \frac{1}{\cos(\pi)} = \frac{1}{-1} = -1$

$\lim\limits_{x\to \pi^+} f(x) = \lim\limits_{x\to \pi^+} \frac{42}{x(x-4)} = \frac{42}{\pi(\pi-4)} \neq -1$. Since the two one-sided limits are not the same we can say $\lim\limits_{x\to\pi} f(x)$ does not exist and thus f is NOT continuous at $x = \pi$.

$x = 0$ We already checked this point, but please note that f IS defined at 0 because we use $\frac{x}{(x-5)(x+6)}$ instead of $\frac{42}{x(x-4)}$ due to the domain. f is NOT continuous because of the limit.

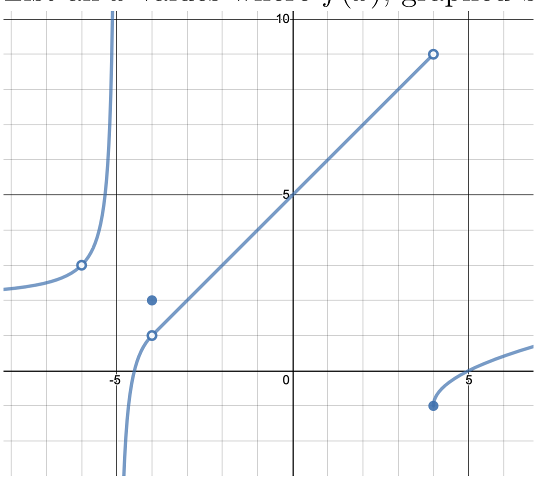
In summary, $f(x)$ is discontinuous at x= -6, $\pi/2$, 0, 4, π.

Prepare for Reassessment:

First, reflect on your mistake and the correct solution and what you learned: fill in the blanks "I used to think ________________but now I think ________________because I learned ________________."

1. Sketch and label a graph that has all 4 types of discontinuities.

2. List all x values where $f(x)$, graphed below, is discontinuous:



3. Find all values of x where the function is discontinuous or explain why the function is always continuous.

   (a) $f(x) = \begin{cases} \ln(x) + 2 & x > 1 \\ 2 & x = 1 \\ (x-1)^2 + 2 & x < 1 \end{cases}$

   (b) $f(x) = \begin{cases} \frac{x-9}{x-3} + 2 & x > 1 \\ 4 & x = 1 \\ (x-1)^2 + 3 & x < 1 \end{cases}$

   (c) $f(x) = \begin{cases} \frac{x-9}{x+3} + 2 & x > 1 \\ 4 & x = 1 \\ \frac{3x}{x(x+2)} & x < 1 \end{cases}$