Find $\lim\limits_{x \to 3} f(x)$ if $f(x) = \begin{cases} x^2 - 1 & \text{if } x \geq 3 \\ x + 2 & \text{if } x < 3 \end{cases}$

Bob the Iguana gives this solution:

$$\lim_{x \to 3} f(x) = \lim_{x \to 3} x^2 - 1 = 3^2 - 1 = 8$$

LeAnn knows that a limit is a value that can be approximated as accurately as desired by choosing input values close enough to, but not equal to, $x=a$. In this problem, $x=3$. So, If Bob is right, then LeAnn should be able to get close to 8 by choosing inputs close to 3.

Try getting within 1 of 8, or between 7 and 9.

try $x = 3.1$: $f(3.1) = 3.1^2 - 1 = 8.61$ and $7 < 8.61 < 9$

try 2.9: $f(2.9) = 2.9 + 2 = 4.9$ This is too small!

get closer to 3?

$f(2.9999) = 2.9999 + 2 = 4.9999$, still too small!

(can we ever get close enough to 3 from the left to get near 8?)

LeAnn is convinced that the limit is not 8.

She explains that because the function changes definitions at $x=3$, and because we are taking the limit as x approaches 3, we need to use one-sided limits.

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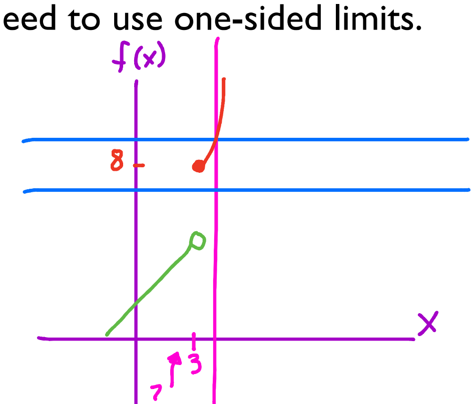
Solution:

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} x + 2 = 3 + 2 = 5$$

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} x^2 - 1 = 3^2 - 1 = 8$$

Because $5 \neq 8$, $\lim\limits_{x \to 3^-} f(x) \neq \lim\limits_{x \to 3^+} f(x)$

therefore $\lim\limits_{x \to 3} f(x)$ does not exist.



Can't make an interval about $x = 3$ that guarantees $f(x)$ is close enough to 8.