Module 15 Quiz Preparation Let u=__________________
Standards: I2, I3, I4, I5, I6, S5 MATH 160 section:

Standards Assessed	**Question**	**Am I ready?**
I2: I can interpret the meaning of an integral in context and recognize when an integral would be suitable to model a real-life situation.	Q1	
I3: I can evaluate a definite integral exactly using geometry or the FToC.	Q2	
I4: I can use shortcuts to compute antiderivatives for sums, constant multiples, and power, polynomial, trig, exponential functions and 1/x.	Q2	
I5: I can evaluate integrals using the substitution method.	Q2	
I6: I can use integrals and antiderivatives to solve differential equations and initial value problems.	Q3	
S5: I can analyze the accumulation function numerically and graphically and compute its derivative.	Q4	

Your quiz will be in class on Friday. Practice not until you get it right, but until you cannot get it wrong! If you can do these questions easily and comfortably, you should be ready for the questions on the quiz. The quiz will not be exactly like these questions, but the skills you use on this document will be skills you will need on the quiz. If you need more practice beyond this document and WeBWorK, look at the Desmos activities, then have some conversations in the Calculus Center.

The questions in this document should help prepare you for the quiz. Make sure to over prepare- quiz questions could vary significantly from these questions, but if you can do these questions, you should be able to handle the quiz.

Resources Allowed: You may consult your notes, any materials from class and the textbook, classmates, tutors and the internet. You are encouraged to visit the Calculus Center and work with classmates. You are not authorized to upload this document or images of this document to any online tutoring service or AI platform.

WeBWorK Practice to prepare for quiz:

I2 WeBWorK #7-8

I3 WeBWorK #3, 4, 8-10, 12, (also 18, 19, 21)

I4 WeBWorK #9-13

I5 WeBWorK #2, 7, 11, 15, 16

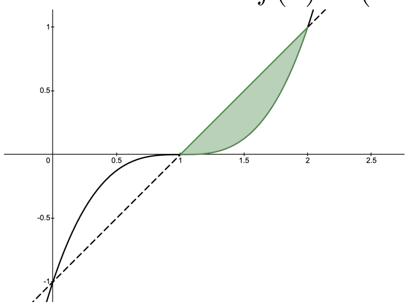
I6 WeBWorK #4-6

S5 WeBWorK #2-7

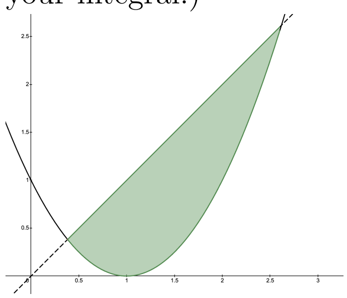
1. *I2: I can interpret the meaning of an integral in context and recognize when an integral would be suitable to model a real-life situation.*

   (a) Set up but do not evaluate, the integral that would give the described or shaded areas:

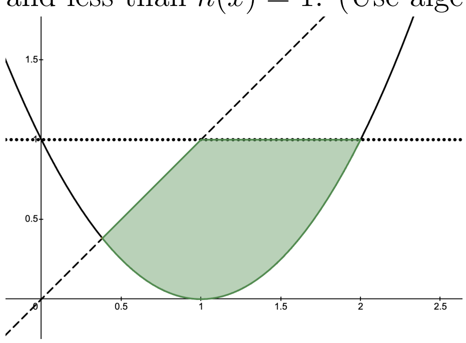
      i. The area between $f(x) = (x-1)^3$ and $g(x) = x - 1$ and above the x-axis:

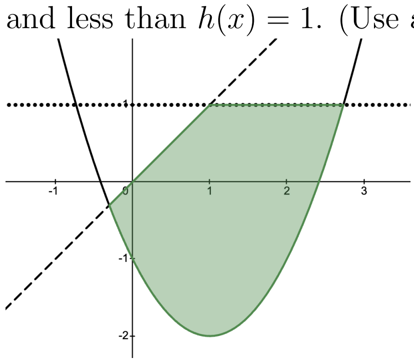
      ii. The area enclosed by $f(x) = (x-1)^2$ and $g(x) = x$. (Use algebra to get the bounds for your integral.)

      iii. The area of the region such that y is greater than $f(x) = (x-1)^2$, less than $g(x) = x$ and less than $h(x) = 1$. (Use algebra to get the bounds for your integrals.)

      iv. The area of the region such that y is greater than $f(x) = (x-1)^2 - 2$, less than $g(x) = x$ and less than $h(x) = 1$. (Use algebra to get the bounds for your integrals.)

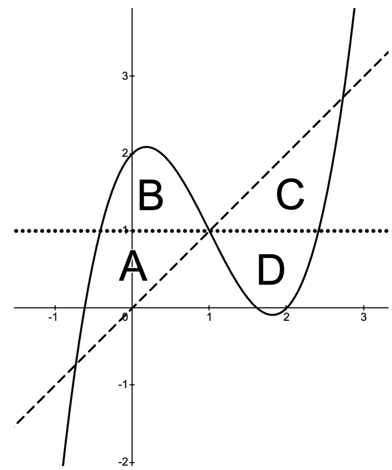
      

(b) Each integral will give the area of a region shown in the graph. Write the corresponding letter or letters of the regions next to each integral.

i. $\displaystyle\int_{-\sqrt{2}+1}^{1} ((x-1)^3 - 2x + 3) - 1)\ dx$

ii. $\displaystyle\int_{1}^{\sqrt{2}+1} (x-1)\ dx +$

$\displaystyle\int_{\sqrt{2}+1}^{\sqrt{3}+1} (x - ((x-1)^3 - 2x + 3))$



iii. $\displaystyle\int_{1}^{\sqrt{2}+1} (1 - ((x-1)^3 - 2x + 3))\ dx$

iv. $\displaystyle\int_{-\sqrt{3}+1}^{1} ((x-1)^3 - 2x + 3) - x)\ dx$

v. $\displaystyle\int_{-\sqrt{3}+1}^{\sqrt{3}+1} |((x-1)^3 - 2x + 3) - 1)|\ dx$

vi. $\displaystyle\int_{-\sqrt{3}+1}^{\sqrt{3}+1} ((x-1)^3 - 2x + 3) - 1)\ dx$

2. *I3: I can evaluate a definite integral exactly using geometry or the FToC.*
I4: I can use shortcuts to compute antiderivatives for sums, constant multiples, and power, polynomial, trig, exponential functions and 1/x.
I5: I can evaluate integrals using the substitution method.

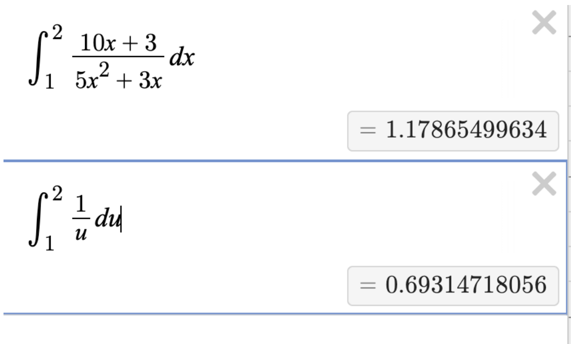
Grading: To get I3, you have to demonstrate that you can use the FToC (antiderivative evaluated at the bounds of the integral.) To get I4, you need to use the correct antiderivative. To get I5, you must show you can solve a definite integral using substitution. This includes treating the bounds of the integral correctly. (The most common mistake is mistreating the bounds and writing something incorrect, or getting the wrong answer because x bounds were plugged in for u or vice versa.)

(a) Hugh Manatee is working on a substitution problem and uses $u = 5x^2 + 3x$ and $du = (10x + 3)dx$. He writes the following:

$$\int_1^2 \frac{10x+3}{5x^2+3x}\ dx = \int_1^2 \frac{1}{u}\ du$$

What is wrong with the above statement? If you aren't sure, enter each side of the equation into Desmos- what do you observe? You might also consider graphing the integrands and look at both integrands on the interval [1,2]. (If you make this kind of mistake you will get NY on standard I5.)

(b) When you do the Webwork, make sure you practice writing out all the steps- your quiz will be grading your work, not just the final answer. The most common reason students miss this question is related to the bounds. You can check for equivalency at each step in desmos: for example: (Hint: type "int" and desmos will turn that into an integral symbol.)



From this you can clearly see a mistake was made and these two integrals as written are NOT equivalent and thus should not be written with "=" between them, as Hugh did above.
What should the bounds (a and b) be so that $\displaystyle\int_a^b \frac{1}{u}\, du = 1.17865$?

(c) Evaluate the definite integrals below using substitution. Pay very close attention to how you handle the bounds! Use Desmos to check the final answer and any intermediate steps!

- $\displaystyle\int_1^5 \frac{3x}{\sqrt{5x^2+7}}\, dx$
- $\displaystyle\int_0^{\pi/4} \sin(x)e^{\cos(x)}\, dx$
- $\displaystyle\int_{\pi/4}^{\pi/3} \sin(x)\cos(x)\, dx$
- Write out the work for all the suggested Webwork practice for I5 as well!

The most common mistakes on problems like these is not using substitution (check your antiderivative with a derivative!). For the last part, make sure you never wrote a definite integral with x values in the bounds but a different variable, like u or w in the integrand.

3. *I6: I can use integrals and antiderivatives to solve differential equations and initial value problems.*

Remember that you can translate questions like $\frac{df}{dx} = 5x^2$ as "What is a function, $f(x)$, that has $5x^2$ as its derivative?". This should help you remember that the final answer is either a family of functions $(+C)$ or, if there is also an initial condition, one specific function.

The most common mistakes on this standard:

- Not using antiderivatives.
- Forgetting $+C$.
- Answering with the C value instead of the entire function. Make sure your final answer is ONE *function.*

(a) Verify that $y = \sin(x) + 3$ is a solution to $y'' = -y + 3$.

(b) Hugh Manatee is solving the initial value problem $\frac{df}{dx} = 5x^2$; $f(1) = 5$.

i. Hugh uses antiderivative shortcuts and gets $f(x) = \frac{5}{3}x^3$. Why isn't Hugh's answer complete?

ii. He shares his work with Bob the Iguana. Bob got $f(x) = \frac{5}{3}x^3 + C$. Why isn't Bob's answer complete?

iii. Dee Rivative shares her answer, saying she used $f(1) = 5$ to get $C = \frac{10}{3}$. Hugh agrees the $+C$ is needed, but thought $C = 5$. Who has the correct value for C?

iv. Bob now thinks the final answer is $C = \frac{10}{3}$, and he draws a box around that answer. Why is he not going to earn S for that answer?

v. What is the full solution to $\frac{df}{dx} = 5x^2$; $f(1) = 5$? Notice your final answer is a function of the form $f(x) = ...$ Check your work by making sure the derivative of your answer is $5x^2$ and that when you plug 1 in for x, you get 5 out.

(c) Consider this initial value problem:

$$\frac{dR}{dq} = \sin(3q) + \cos(3q) \qquad R(\pi/3) = a$$

i. The solution to the differential equation will be a function. What letter should be used for the output variable? What letter should be used for the input variable?

ii. Write the most general solution to the differential equation. "Most general" means that your answer needs to represent the entire family of functions that solve this differential equation.

iii. Let $a=$ the last 4 digits of your CSU ID number.

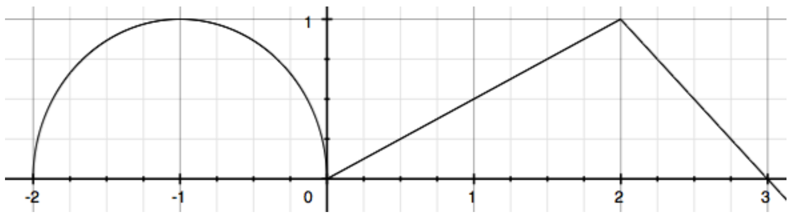
$$R(\pi/3) = a =$$

iv. Use the initial condition (with your a from part c) to write the particular solution to the initial value problem. Make sure that your final answer is the **function** that solves the initial value problem, not a just number.

4. *S5: I can analyze the accumulation function numerically and graphically and compute its derivative.*

   (a) Consider the function $g(x)$ defined by $g(x) = \int_0^x f(t)dt$.

   The graph of f is given below. Note that on the interval $[-2, 0]$ the graph is a semicircle.



   Answer the following questions and explain how you got your answers.

   i. Evaluate: $g(-2)$, $g(-1)$, $g(0)$, $g(1)$, $g(2)$, and $g(3)$

   ii. Evaluate: $\left.\frac{dg}{dx}\right|_{x=2}$, $g'(-1)$, $g'(1)$, $g'(3)$ and $\left.\frac{dg}{dx}\right|_{x=0}$

   iii. Evaluate: $g''(-1)$, $g''(0)$, $g'''(1)$, $g''(2)$, and $g''(2.5)$

   iv. List all the critical points of $g(x)$. Does $g(x)$ have a local maximum, minimum or neither when $x = 3$?

   v. Find values for a and b so that $\int_a^b f(x)\,dx = -1/2$

(b) Compute the following derivatives:

i. $\dfrac{d}{dx} \displaystyle\int_2^x e^{-t}\, dt$

ii. $\dfrac{d}{dx} \displaystyle\int_0^{\sin(x)} \cos(t)\, dt$

iii. $\dfrac{d}{dx} \displaystyle\int_3^{3x^2} \frac{1}{t^2+t}\, dt$

iv. $\dfrac{d}{dx} \displaystyle\int_3^{x^4+3} \sqrt{t^2-4}\, dt$