

**Math 651, Assignment 5**  
**Due Friday, November 17**

**Overview**

In this assignment, you will program the quasi-Newton algorithm developed in class and then use your program to explore a series of test problems. Your program should have the option of using a global line search strategy or pure Newton iteration (both using an approximate finite difference Jacobian) and should provide an estimate of the accuracy of the last iterate as well as details of the convergence of the algorithm. You should compare the behavior of the quasi-Newton and full Newton iterations on each problem.

**Assignment Report**

Your solution for this assignment should be written in the style of a laboratory report with an introduction that explains the program you use (a well-commented listing of your program would suffice); and then sections on each test problem giving the results and any explanation/analysis of the results you can make. Your report should include a summary that explains why and how you feel your algorithm works based on the results of the test problem. To be convincing, the summary should include at least one convergence analysis in which you demonstrate that the iterates converge at a second order rate.

**Test Problems**

1. This is an exploration of the effect of the difference step in the approximate Jacobian on the accuracy of the computed root. Apply your algorithm to compute the root of  $f(x) = x^2 - 2$  starting with  $x_0 = 1$  ( $\bar{x} = \sqrt{2}$ ) using a sequence of step sizes in the computation of the approximate Jacobian equal to  $h_i = 10^{-i} \times \max\{|x_{m-1}|, 1\} \times \text{sign}(x_{m-1})$ , where  $x_{m-1}$  is the previous iterate in the  $m^{\text{th}}$  step, for  $i = 1, \dots, 15$ . (This will require a temporary modification of your program.) Report on the convergence behavior of the algorithm and the accuracy of the computed roots as  $h_i$  varies.
2.  $f = (x_1^2 + x_2^2 - 2, e^{x_1-1} + x_2^3 - 2)^\top$ ,  $x_0 = (2, .5)^\top$ , and  $\bar{x} = (1, 1)^\top$ .
3.  $f = (10(x_2 - x_1^2), 1 - x_1)^\top$ ,  $x_0 = (-1.2/\alpha, \alpha)^\top$  for  $\alpha = .01, .1, 1, 10, 100$ ,  $\bar{x} = (1, 1)^\top$ .

4. Given integer  $n$ ,  $f_i(x) = n - \sum_{j=1}^n (\cos(x_j) + i(1 - \cos(x_i)) - \sin(x_i))$ ,  
 $x_0 = (1/n, 1/n, \dots, 1/n)^\top$ , and the desired root is  $\bar{x} = (0, \dots, 0)^\top$ .  
 Use  $n = 5$ .
5.  $f = (10(x_3 - 10\Theta(x_1, x_2)), 10((x_1^2 + x_2^2)^{1/2} - 1), x_3)^\top$  with  $\Theta(x_1, x_2) =$   
 $\frac{1}{2\pi} \tan^{-1}(x_2/x_1)$  if  $x_1 > 0$  and  $\Theta(x_1, x_2) = \frac{1}{2\pi} \tan^{-1}(x_2/x_1) + .5$  if  
 $x_1 < 0$ ,  $x_0 = (-1, 0, 0)^\top$ ,  $\bar{x} = (1, 0, 0)^\top$
6.  $f = x((x - 4)^2 + 1)^{1/2}$ ,  $x_0 = 4.1$ ,  $\bar{x} = 0$ .
7.  $f = \cos(x) + \sin^2(50x)$ ,  $x_0 = \pi/2 - .1$