Exam 2 **Page 1** **Name:** Key

1. Solve the initial value problem $y'' + y' - 6y = 0$ with $y(0) = 0$ and $y'(0) = -1$.

char. eq. $r^2 + r - 6 = 0$ 6pts $(r+3)(r-2)=0$

$r \quad \frac{-1 \pm \sqrt{1+24}}{2} = \frac{-1 \pm 5}{2} \rightarrow$

$r_1 = -3 \qquad r_2 = 2$ 4pts

$y = A_1 e^{-3x} + A_2 e^{2x}$ 6pts

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$y(0) = A_1 + A_2 = 0 \qquad \underline{A_2 = -A_1} \quad 4$

$y' = -3A_1 e^{-3x} + 2A_2 e^{2x}$

$y'(0) = -3A_1 + 2A_2 = \underline{-1 \qquad 4}$

$-3A_1 - 2A_1 = -1$

$A_1 = \frac{1}{5}$

$A_2 = -\frac{1}{5}$ 6

$y = \frac{1}{5} e^{-3x} - \frac{1}{5} e^{2x}$ 14pts

30pts

2. Solve the initial value problem $y'' + 4y' + 4y = 0$ with $y(1) = 0$ and $y'(1) = 2e^{-2}$

Char. eq. $r^2 + 4r + 4 = 0$ 6 pts

$(r+2)^2 = 0$

$r_1 = r_2 = -2$ 4 pts

$y = A_1 e^{-2x} + A_2 x e^{-2x}$ 6 pts

$y(1) = A_1 e^{-2} + A_2 e^{-2} = 0$

$A_1 = -A_2$

$y'(x) = -2A_1 e^{-2x} + A_2 e^{-2x} - 2A_2 x e^{-2x}$

$y'(1) = -2A_1 \cancel{e^{-2}} + A_2 \cancel{e^{-2}} - 2A_2 \cancel{e^{-2}} = 2\cancel{e^{-2}}$

$= -2A_1 - A_2 = 2$

so $2A_2 - A_2 = 2 \Rightarrow A_2 = 2$

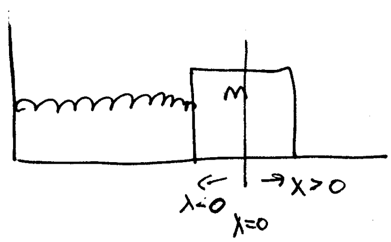
$A_1 = -2$

$y = -2e^{-2x} + 2x e^{-2x}$ 14 pts

$= (-2 + 2x) e^{-2x}$

30 pts

3. A detailed measurement of a spring shows that the force exerted by the spring when it is stretched/compressed a distance from the equilibrium is given by the sum of a term proportional to the distance stretched and a term proportional to the cube of the distance stretched. Both terms resist the motion. Write down a differential equation describing the motion of a mass-spring system where the mass rests on a flat frictionless surface and the spring is connected to the mass at one end and a fixed wall at the other end. Draw a picture illustrating the system and identify the variable used in the differential equation. Be sure to identify all constants used in the equation **and their signs as well.**



$$m\ddot{x} = -k_1 x - k_2 x^3$$

$k_1, k_2 > 0$ 10pts

-3 equal k

-1, -2, -3 no signs

4. Write down a **particular solution** of the problem $y'' - 3y' + 2y = 6e^{4x}$.

Guess $y_p = Ce^{4x}$ 6pts

$y_p' = 4Ce^{4x}$

$y_p'' = 16Ce^{4x}$

$y_p'' - 3y_p' + 2y_p = 16Ce^{4x} - 12Ce^{4x} + 2Ce^{4x}$

$= 6Ce^{4x} = 6e^{4x}$ all x

$\underline{C = 1}$ 9pts

$y_p = e^{4x}$

No $C \Rightarrow -10$

15pts

5. Given $y(t) = c_1 e^t \cos(t) + c_2 e^t \sin(t)$, what is the second order, constant coefficient, homogeneous differential equation that has y as a general solution. Hint: determine the characteristic equation.

roots: $(1 + 1 \cdot i), (1 - 1 \cdot i)$

$\rightarrow y \; c_1 e^t \cos(1 \cdot t) + c_2 e^t \sin(1 \cdot t)$

char. equation $(r + (1+i))(r - (1-i))$ ‖ 6

$= r^2 - (1+i)r - (1-i)r + (1+i)(1-i)$

$= r^2 - r - \cancel{ir} - r + \cancel{ir} + 1 + 1$

$= r^2 - 2r + 2$ check $r = \frac{2 \pm \sqrt{4-8}}{2}$ ‖ 5

$= 1 \pm i$ ✓

So

$y'' - 2y' + 2y = 0$ ‖ 4

= -3

15pts