

# Introduction to Differential Manifolds (Math 670)

Spring 2015

## Technicalities

**Instructor:** Dr. Clayton Shonkwiler ([clayton@math.colostate.edu](mailto:clayton@math.colostate.edu))

**Office:** Weber 216

**Course web page:** <http://www.math.colostate.edu/~clayton/teaching/m670s15/>

**Time/Location:** 11:00–11:50 MWF, Engineering E205.

**Office Hours:** Wednesday 1:00–3:00, Thursday 1:00–2:00, and by appointment.

## Summary of the Course

The course will be an introduction to differentiable manifolds with an eye towards Lie groups and homogeneous spaces, especially Stiefel manifolds, Grassmannians, and other flag manifolds.

We will start with the basics of differentiable manifolds (tangent spaces, vector fields, Lie brackets, etc.) and come to grips with differential forms and tensors. With that background under our belts we will be able to dive into the study of Lie groups. Lie groups are groups which are also manifolds – such as matrix groups like  $GL(n, \mathbb{R})$  and  $SU(n)$  – and they are central to much of modern mathematics.

Finally, we will study homogeneous spaces, which can be interpreted as quotients of Lie groups or, alternatively, as highly symmetric spaces equipped with large group actions. Stiefel, Grassmann, and flag manifolds are examples of homogeneous spaces which pop up again and again in both pure and applied mathematics, from data analysis to algebraic geometry to combinatorics to random walks; to the extent that time permits, the last part of the course will be devoted to developing connections between the geometry of these spaces and other areas in mathematics.

Some familiarity with point-set topology, multivariable calculus, and linear algebra will be very helpful, but otherwise the course should be reasonably self-contained.

There is no official text for the course, but the following books may be useful resources:

- *Foundations of Differentiable Manifolds and Lie Groups*, by Frank W. Warner
- *Calculus on Manifolds* and *A Comprehensive Introduction to Differential Geometry*, by Michael Spivak
- *Notes on Differential Geometry and Lie Groups*, by Jean Gallier (only available online at <http://www.cis.upenn.edu/~jean/gbooks/manif.html>)
- *Differential Topology*, by Victor Guillemin and Alan Pollack
- *Differential Geometry, Lie Groups and Symmetric Spaces*, by Sigurdur Helgason

## 1 Grading

This is an advanced graduate course, so your grade will be based on homework and a final project:

**Homework:** 60%

**Final Project:** 30%

**Class Participation:** 10%

Homework will be collected regularly throughout the semester. For the final project, you will investigate a connection between material from this class and some other area of mathematics (combinatorics, probability, applied math, etc.) and write a short paper and give a 20-minute talk about this connection.

## 2 Disclaimer

The course syllabus is a general plan for the course; deviations announced in class may be necessary.

## 3 Anticipated Schedule

<b>Topic</b>	<b>Weeks</b>
Manifolds and Vector Fields	2
Differential Forms	3
Lie Groups	3
Homogeneous Spaces	2
Applications and Connections	3