**Math 369 Exam #1 Practice Problems**

1. Find the set of solutions of the following system of linear equations. Show enough work to make your steps clear.

$$\begin{aligned} x + 2y + 3z + 4w &= 1 \\ -2x - 3y - 4z - 6w &= 1 \\ 3x + 5y + 7z + 10w &= 0 \end{aligned}$$

2. (a) Suppose $a, b, c$ are nonzero numbers. Find the inverse of the matrix $\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$.

   (b) Find the inverse of the matrix $\begin{pmatrix} 1 & d & e \\ 0 & 1 & f \\ 0 & 0 & 1 \end{pmatrix}$, where $d, e, f$ are any real numbers.

   (c) True or false: *Every upper triangular matrix with nonzero diagonal entries has an inverse.* Explain your answer.

   *Hint:* Every such matrix can be written as the product

$$\begin{pmatrix} a & x & y \\ 0 & b & z \\ 0 & 0 & c \end{pmatrix} = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} \begin{pmatrix} 1 & x/a & y/a \\ 0 & 1 & z/b \\ 0 & 0 & 1 \end{pmatrix}$$

3. Let $A \in \mathrm{Mat}_{2,3}(\mathbb{R})$ be a $2 \times 3$ matrix.

   (a) Let $U = \{\vec{x} \in \mathbb{R}^3 : A\vec{x} = \vec{0}\}$. Show that $U$ is a subspace of $\mathbb{R}^3$.

   (b) Is $W = \{\vec{x} \in \mathbb{R}^3 : A\vec{x} = \vec{b}\}$ a subspace when $\vec{b} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$? Explain.

4. Let $V$ be a vector space.

   (a) Define what it means for a set $\{u_1, \ldots, u_n\} \subset V$ to be linearly dependent.

   (b) Suppose $v \in V$. Is the set $\{0, v\}$ linearly dependent? Explain.

5. Let $V = \mathcal{P}(\mathbb{R})[z]$ be the vector space of polynomials with real coefficients in the variable $z$. Is the set

$$\{1 + z + z^2, 1 - z, 1 - z^3\}$$

   linearly independent? Prove your claim.