Math 3200 Exam #3 Practice Problems

1. Suppose $[a], [b] \in \mathbb{Z}_6$ so that $[a] \cdot [b] = [0]$. Can you conclude that either $[a] = [0]$ or $[b] = [0]$? Why or why not?

2. List all the possible equivalence relations on the set $A = \{a, b\}$. (For organizational purposes, it may be helpful to write the relations as subsets of $A \times A$.)

3. Define the relation R on $\mathbb{Z}$ by $x\,R\,y$ if $x^2 \equiv y^2 \pmod 4$. Is R an equivalence relation? If so, what are the equivalence classes of R?

4. Define the relation R on $\mathbb{R}$ by $x\,R\,y$ if $xy > 0$. Is R an equivalence relation? If so, what are the equivalence classes of R?

5. Suppose R_1 and R_2 are equivalence relations on a set A. Define the relation R on A by $x\,R\,y$ if $x\,R_1\,y$ and $x\,R_2\,y$. Is R an equivalence relation?

6. Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c\}$. Find a function $f : A \to B$ which is either injective or surjective, but not both.

7. Define the function $g : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ by $f(m, n) = 2n - 4m$.

   (a) Is g injective? Prove or give a counterexample.

   (b) Is g surjective? Prove or give a counterexample.

8. Define the function $h : \mathbb{Z}_8 \to \mathbb{Z}_8$ by
$$h([a]) = [a^3].$$

   (a) Is h injective? Prove or give a counterexample.

   (b) Is h surjective? Prove or give a counterexample. (*Hint: Why does your answer to part (a) provide the answer to this question without doing any additional work?*)

9. Suppose A, B, and C are sets and that $f : A \to B$ and $g : B \to C$ are functions. If $g \circ f$ is surjective, is f necessarily surjective? Prove or give a counterexample.

10. Define the sequence $a_1, a_2, a_3, \ldots$ by
$$a_1 = 1, \quad a_2 = 2, \quad \text{and} \quad a_n = 2a_{n-1} - a_{n-2} \text{ for all } n \geq 3.$$
Prove that $a_n = n$ for all $n \in \mathbb{N}$.