Math 2260 Final Exam Practice Problems

1. Write down an integral which will compute the length of the part of $y = \ln x$ for $1 \le x \le 2$. Don't worry about evaluating this integral.

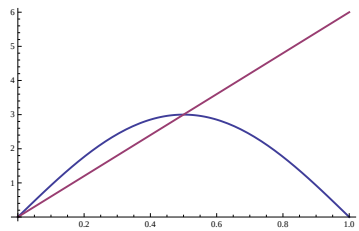
2. Let $V(a)$ be the volume obtained by rotating the area between the x-axis and the graph of $y = \frac{1}{x^{3/2}}$ from $x = a$ to $x = 1$ around the y-axis. What happens as a goes to zero?

3. What is the volume of the solid obtained by rotating the part of the graph of $y = \cos x\sqrt{\sin x}$ between $x = 0$ and $x = \pi/2$ around the x-axis?

4. Find the volume obtained by rotating the region between the x-axis, the y-axis, and the line $x + y = 1$ around the line $x = -2$.

5. Calculate the surface area obtained by rotating $y = \sqrt{x}$ around the x-axis for $3/4 \le x \le 15/4$.

6. (a) Find the area between $y = 3\sin(\pi x)$ and $y = 6x$ for $x > 0$.



   (b) What is the volume obtained by rotating this region around the x-axis?

   (c) What is the volume obtained by rotating around the y-axis?

7. What is the volume obtained by rotating the region between $y = \frac{1}{\sqrt{x^2+4}}$ and the x-axis for $0 \le x \le 3$ around the x-axis?

8. What is the length of the part of $y = x^{3/2}$ from $x = 1$ to $x = 6$?

9. Integrate $\int x^4 \ln(2x)\, dx$.

10. Integrate $\int x^2 \cos(3x)\, dx$.

11. Integrate $\int \frac{x+2}{x^2+8x+15}\, dx$.

12. Integrate $\int \sqrt{1 - 4x^2}\, dx$.

13. Integrate $\int \frac{\sin^2(\ln(x))}{x}\, dx$.

14. Integrate $\int \frac{e^t}{e^{2t}+4}\, dt$.

15. Evaluate $\int_e^\infty \frac{1}{x(\ln x)^2}\, dx$.

16. Evaluate $\int_0^\infty \frac{dx}{(x+1)(x+4)}\, dx$.

17. Solve the initial-value problem
$$\frac{dy}{dx} + y = 2, \quad y(0) = 1.$$

18. Solve the initial-value problem
$$\frac{dy}{dx} = 1 - y + x^2 - yx^2, \quad y(0) = 0.$$

19. Consider the sequence $(a_n)_{n=1}^{\infty}$, where

$$a_n = \frac{\ln(1+3/n)}{\sin(2/n)}.$$

Does the sequence converge? If so, to what? If not, explain why.

20. Consider the sequence $(a_n)_{n=1}^{\infty}$, where

$$a_n = \left(\frac{n+1}{n+2}\right)^n.$$

Does the sequence converge? If so, to what? If not, explain why.

21. Which of the following series converge?

    (a) $\sum_{n=1}^{\infty} \frac{n^3}{3^n}$
    (b) $\sum_{n=1}^{\infty} \frac{n^3}{1+n^4}$
    (c) $\sum_{n=1}^{\infty} \frac{n^3}{1+n^5}$
    (d) $\sum_{n=1}^{\infty} \frac{\cos^2 n}{1+e^{-n}}$
    (e) $\sum_{n=1}^{\infty} \frac{e^n}{n!}$

22. If you put $(-1)^n$ into each of the series in the preceding problem, which ones converge absolutely, which converge conditionally, and which diverge?

23. Does the series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln(n))^2}$$

converge absolutely, converge conditionally, or diverge?

24. Find the interval of convergence for each of the following power series.

    (a) $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n}$
    (b) $\sum_{n=1}^{\infty} \frac{(x-3)^n}{1+n^2}$
    (c) $\sum_{n=1}^{\infty} \frac{(-1)^n(x-1)^n}{ne^n}$
    (d) $\sum_{n=1}^{\infty} n^2 x^n$

25. Find the Taylor series for

    (a) e^{-x^2} centered at $x = 0$
    (b) $\sqrt{x}$ centered at $x = 1$

26. Estimate $e^{-0.2}$ to the nearest 0.001. Explain your answer.

27. Does the series

$$\sum_{n=1}^{\infty} \frac{n^7 6^n}{n!}$$

converge or diverge? Explain your answer.

28. Does the series

$$\sum_{n=2}^{\infty} \frac{1}{n+\sqrt{n}}$$

converge or diverge? Explain your answer.

29. Find the limit

$$\lim_{x\to 0} \frac{\cos x - 1}{e^{x^2} - 1}.$$

30. Give a geometric description of the set of points (x, y, z) satisfying the following inequalities.

    (a) $x^2 + y^2 + z^2 \leq 1$ and $z \geq 1/2$.

    (b) $|x| \leq 1$, $|y| \leq 1$, and $|z| \leq 1$.

31. Determine the angle between the vectors

$$\vec{\mathbf{u}} = \vec{\imath} + 3\vec{\jmath} + 2\vec{\mathbf{k}} \qquad \text{and} \qquad \vec{\mathbf{v}} = -2\vec{\imath} + \vec{\jmath} - 2\vec{\mathbf{k}}$$

32. Write the vector

$$\vec{\mathbf{u}} = \vec{\imath} + \vec{\jmath} + 5\vec{\mathbf{k}}$$

as the sum of two vectors, one parallel to $\vec{\mathbf{v}} = -2\vec{\imath} - \vec{\jmath} - \vec{\mathbf{k}}$ and one perpendicular to $\vec{\mathbf{v}}$.

33. Find the area of the parallelogram spanned by the vectors

$$\vec{\mathbf{u}} = -\vec{\imath} + \vec{\jmath} - \vec{\mathbf{k}} \qquad \text{and} \qquad \vec{\mathbf{v}} = 3\vec{\imath} + 2\vec{\jmath} + \vec{\mathbf{k}}.$$