

Math 2250 Final Exam Practice Problems

1. What are the domain and range of the function

$$f(x) = \frac{\ln x}{\sqrt{x}}?$$

2. Find the inverse of the function $f(x) = 1000(1 + 0.07)^x$.
3. Find the point on the graph of $y = e^{3x}$ at which the tangent line passes through the origin.
4. Find the equation of the tangent line to the curve

$$xy^3 - x^2y = 6$$

at the point $(3, 2)$.

5. Use an appropriate linearization to approximate $\sqrt{96}$.
6. Consider the function $f(x) = x^2e^{-x^2}$. What is the absolute maximum of $f(x)$?
7. A movie theater has been charging \$7.50 per person and selling about 400 tickets on a typical weeknight. After surveying their customers, the theater estimates that for every \$1.50 that they lower the price, the number of moviegoers will increase by 30 per night. This means the graph of the demand function $p(x)$ is a line passing through the points $(400, 7.5)$ and $(430, 6)$; using the point-slope formula, this means that $p(x) = -\frac{x}{20} + 27.5$. Find the price which will maximize the theater's revenue.
8. Water is draining from a conical tank at the rate of 18 cubic feet per minute. The tank has a height of 10 feet and the radius at the top is 5 feet. How fast (in feet per minute) is the water level changing when the depth is 6 feet? (Note: the volume of a cone of radius r and height h is $\frac{\pi r^2 h}{3}$.)
9. The function $f(x) = x^4 - 6x^3$ is concave down for what values of x ?
10. Evaluate the limit

$$\lim_{x \rightarrow 0} (1 - 6x)^{1/x}.$$

11. Let $f(x) = x^{\cos x}$. What is $f'(\pi/2)$?
12. For $0 \leq t \leq 5$, a particle moves in a horizontal line with acceleration $a(t) = 2t - 4$ and initial velocity $v(0) = 3$.
(a) When is the particle moving to the left?
(b) When is the particle speeding up?
(c) What is the position of the particle at time t if the initial position of the particle is 6?
13. If $\int_0^6 f(x)dx = 10$ and $\int_0^4 f(x)dx = 7$, find $\int_4^6 f(x)dx$.

14. Evaluate the definite integral

$$\int_{\pi/6}^{\pi/4} \sin t \, dt.$$

15. Evaluate the integral

$$\int \frac{2}{t-3} \, dt.$$

16. Evaluate the definite integral

$$\int_1^4 \frac{2\sqrt{x} + 4x^2}{x} dx$$

17. Suppose the velocity of a particle is given by

$$v(t) = 6t^2 - 4t.$$

What is the displacement (i.e. change in position) of the particle from $t = 0$ to $t = 2$?

18. Suppose that

$$\int_0^{x^2} f(t) dt = \sqrt{x^2 + 1} - 1.$$

What is $f(2)$?

19. Evaluate the integral

$$\int 3e^{\tan x} \sec^2 x dx.$$

20. Evaluate the definite integral

$$\int_0^{\pi/16} 8 \tan(4x) dx.$$

21. What is the area of the red region in the figure? The blue curve is given by $y = 2\frac{\ln x}{x}$ and the vertical green lines are the lines $x = \frac{1}{e}$ and $x = e$.

