For these questions, use Matlab or equivalent. Don’t forget to adjust for the fact that Matlab sequences start with index 1, whereas our sequences start with index 0.

1. For the sequence $f_j = \sin(k_0 x_j)$ where $k_0 = 20$ and $x_j = 2\pi j/N$, $j = 0,1,\ldots,N$ with $N = 128$:
   (a) Plot the sequence $(f_j)$ and its DFT $(F_n)$.
   (b) Zero-pad your original sequence (at the end) to length $4N$, take the DFT, and plot your result. Compare with your original DFT.
   (c) Zero-pad $(F_n)$ in the MIDDLE to length $4N$, take the inverse DFT, and plot your result. Compare with your original $(f_j)$.
   (d) Zero-pad $(F_n)$ at the END to length $4N$, take the inverse DFT, and plot your result. Compare with your original $(f_j)$.

2. For $N = 512$ and $M = N/2$, define the sequences
   $$x_j = \begin{cases} 1 & j = 0,1,2,\ldots,(N/2) - 1 \\ 0 & j = (N/2),\ldots,N-2,N-1 \end{cases}$$
   $$w_j = \frac{1}{2} \left[ 1 - \cos\left( \frac{2\pi j}{M - 1} \right) \right]$$
   Note: The sequence $(w_j)$ is called a Hanning window.
   (a) Plot $(x_j)$ and its FFT $(X_n)$.
   (b) Use the Matlab conv command to convolve $(x_j)$ with the sequence $(w_j)$. What is the length of the resulting vector $(y_j)$? Plot $(y_j)$. What do you notice about the relationship between $(x_j)$ and $(y_j)$?
   (c) Multiply the length-512 FFTs of $(w_j)$ and $(x_j)$ and take the length-512 IFFT of the product. Compare this result with your result from part (b). You should be able to observe aliasing effects.
   (d) Zero-pad the sequence $(w_j)$ at the end, and plot the FFT of the product sequence $(x_jw_j)$. What do you notice about the relationship between this result and the original FFT $(X_n)$?

3. Compute the Lebesgue integral of the function that is 1 on the irrational numbers between 0 and 1 and zero on the rationals.