Homework Set 5
Math/ECE 430
due Friday, March 1, 2013

For these questions, use Matlab or equivalent. Note that Matlab indices start at 1 instead of zero. Use Matlab help to find the formulas Matlab uses for the commands fft and ifft.

1. (This question is from Prof. Laurent Demanet, MIT) Fix $N = 128$. Consider the cosine wave

$$f_j = \cos(k_0 x_j)$$

with $k_0 = 20$ and $x_j = 2\pi j/N$, $0 \leq j \leq N - 1$. Compute its DFT ($F_n$) with the fft command. Plot the result.

(a) For what values is the DFT nonzero?

(b) In view of $\cos a = (e^{ia} + e^{-ia})/2$, what frequencies $k$ do you expect to be present in $\cos(k_0 x_j)$?

(c) Reconcile your answers to (a) and (b).

(d) Apply the fftshift command to the output of the fft and re-plot the result.

(e) Undo the fftshift (by applying it a second time), apply the inverse DFT with ifft, and plot the result to check that you recover ($f_j$). How accurate is the reconstruction?

(f) ($f_j$) is real and even, so its DFT ($F_n$) should also be real and even. Verify this.

2. Compute the DFT of the vector $(1, 1, \ldots, 1)$ with the fft command; plot the result. Apply fftshift and plot the result.

3. Compute the DFT of $(0, 0, 0, 1, 0, \ldots, 0)$ with the fft command; plot the result. Apply fftshift and plot the result.