Homework Set 10

Math/ECE 430

due Friday, April 12, 2013

The page numbers below refer to the book by Gasquet and Witomski. $\mathcal{F}$ denotes the Fourier transform.

1. p. 274, #29.7. Prove that
   \[ \sum_{n=-\infty}^{\infty} (-1)^n e^{i\pi nx} = 2 \sum_{n=-\infty}^{\infty} \delta_{2n+1}. \]
   Hint: Use the Fourier series expansion of the function $x$ on $(-1,1)$.

2. p. 295, #31.5 Show that the Fourier transform of the Heaviside function is $\frac{1}{2\pi i} \text{P.V.}(1/\xi) + \lambda \delta$. Show that $\lambda = 1/2$ by noting that P.V.(1/x) is odd.

3. p. 295, # 31.6. Show that the Fourier transform of P.V. (1/x) is proportional to $\text{sgn} \,(\xi)$, and find the constant of proportionality. Hint: You can compute this directly, without using a test function. Look for the sign function in the change of variables, and use $\int_0^{\infty} (\sin x)/x \,dx = \pi/2$.

4. p. 295, #31.8. Show that
   \[ \mathcal{F}[\cos 2\pi \lambda t] = (\delta_{\lambda} + \delta_{-\lambda})/2 \quad \text{and} \quad \mathcal{F}[\sin 2\pi \lambda t] = (\delta_{\lambda} - \delta_{-\lambda})/2. \]

5. p. 351, #37.1. a) Use the Fourier transform relation for Dirac’s comb to show that
   \[ \sum_{n=-\infty}^{\infty} \phi \left( \frac{n}{a} \right) = a \sum_{n=-\infty}^{\infty} |\mathcal{F}\phi|(na) \quad (a \neq 0) \]
   b) Deduce from this that
   \[ \sum_{n=-\infty}^{\infty} e^{-\pi n^2 t} = \frac{1}{\sqrt{t}} \sum_{n=-\infty}^{\infty} e^{-\pi n^2 / t} \]