

M450: Assignment # 1

August 23rd, 2007

Problem 1

Let $f(x, y) = \ln(1 + x^2 + y^2)$. Compute the linearization of f at $(0, 0)$ (This is the same as the Taylor polynomial of degree 1). Find a bound for the linearization error in max-norm (in terms of h) with $\Omega = (-h, -h) \times (h, h)$. You may want to use the symbolic capabilities of MATLAB to compute the necessary second derivatives.

Problem 2

The expressions e^h , $(1 - h^4)^{-1}$, $\cos(h)$, and $1 + \sin(h^3)$ all have the same limit as $h \rightarrow 0$. Express each in the following form, with optimal choices for α and β .

$$f(h) = c + \mathcal{O}(h^\alpha) = c + (o)(h^\beta)$$

What are the limit and rate of convergence of the following expression as $h \rightarrow 0$?

$$\frac{1}{h^2}[(1 + h) - e^h]$$

Express the limit in the form $c + \mathcal{O}(h^\alpha)$.

Problem 3

Write a program that does the following:

1. Generate a vector of 10^4 random (double) entries between 0 and 1.
2. Produce ascending and descending versions of the vector by sorting.
3. Sum the entries of the 3 “generated” vectors (do this manually (for-loop) and via the **sum** command (if you are using matlab)).
4. Make a symbolic copy of the original vector and perform an exact sum of the entries.

5. Use this exact sum to compute the relative errors of the 3 vector sums. Are your results as expected?

Problem 4

Let $f(x) = 1/x * (1 - \cos(x))$. As $x \rightarrow 0$, $f(x) \rightarrow 0$. Let $x = 10^{-N}$, $N = 1 \dots 12$. Write a code to compute an “approximate” limit (i.e. compute $f(x)$ for the various values of N). “Rewrite” f and repeat to generate a more stable computation.