

## Questions to Ask Yourself to Solve Optimization Problems

Based on: Polya, G. *How to Solve It*, expanded Princeton Science Library edition. Princeton University Press, Princeton and Oxford, 2004.

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### Question 1: Will drawing a diagram help me understand the situation?

(If you think it will, draw a diagram! If you are not sure, draw a diagram!)

### Question 2: What are the data? What information do I have from the statement of the problem?

Optimization problems *always* involve at least two variable quantities. The problem may also involve some constants and, occasionally, other variables. Be careful not to confuse constants and variables.

- **What are the constants in the problem?**
- **What are the variable quantities in this problem. (There is always at least two!)  
In the setting of the problem, what values can these variables have? Can they be negative? very large? very small?**
- **Can you describe a connection among these constants and variables (in words)?**

### Question 3: What is the unknown? What information am I asked to find?

In an optimization problem there is *always* a quantity that you are asked to make as large or small as possible (optimize) by controlling some other a quantity or quantities.

- **What is the quantity you want to optimize? Are you asked to make it as small as possible or as large as possible? Does it seem reasonable that this quantity has an optimum value?**

### Question 4: What notation would help? Introduce suitable notation.

Optimization problems *always* involve a quantity or quantities you can control (independent variables) and a quantity that you want to optimize (dependent variable).

- **What are the independent variables? Is a reference point or coordinate system needed to quantify the independent variable? If so, where should the origin and axes be?**  
Independent variables are those you can control directly. Give the independent variable (or variables if there is more than one) letter names. It is often helpful to choose letters that suggest the quantity represented – e.g.  $r$  for radius and  $h$  for height.
- **What is the dependent variable?**  
The quantity you are to optimize is the dependent variable, Give it a **function** name. Indicate the independent variable(s) as part of the function name. *E.g.*, if you are to minimize the surface area of a container and the surface area is determined by the depth  $d$  of the container, denote the surface area by  $S(d)$ . If your are to minimize a distance that depends on the direction  $\theta$  of travel, denote the distance by  $D(\theta)$ .

### Question 5: Can I express the data (the known information) using the notation I introduced?

**Can I express the unknown using the notation I introduced?**

**Can I state the problem using this notation?**

(If you can't do all these things, reexamine your notation and your understanding of the problem.)

### Question 6: What are some connections between the knowns and the unknown?

**Which of these connections might I use to find the unknown?**

**Can I use the notation I introduced to write an equation for this connection?**

**Can I use this equation to find the unknown?**

Write an equation that expresses the function you want to optimize (the dependent variable) in terms of the other variables in the problem. If necessary, use information about the connections among the independent variables to express this function in terms of the *one* independent variable.

What is a reasonable interval of possible numerical values for the independent variable? Is there a reason why this function should have extreme values on this interval?

Now find the absolute maximum or absolute minimum of this function (as the problem requires).

### Question 7: Does my answer seem reasonable?

**What strategies did I use in solving this problem that I might be able to use in another situation?**

## Scoring Criteria for Optimization Problems

*What we will look for when we grade optimization problems on midterm and final exams.*

- If applicable, a correct figure or diagram with constants and variables labeled using appropriate notation.
- Clear identification of the dependent variable (what is to be optimized) and indication of whether it is to be maximized or minimized.
- A function (given by an equation) that expresses the dependent variable in terms of the one other variable in the problem (the independent variable) including specification of the domain of this function.  
In some problems, the dependent variable is first given in terms of two independent variables. In these problems we look for a clearly stated equation relating the two independent variables that is used to express the dependent variable in terms of only one independent variable.
- Clear presentation of using calculus methods to
  - (i) correctly find the optimum (maximum or minimum) and
  - (ii) verify that the value found is the optimum.

**PROBLEM:** Find the volume of the largest right circular cone that can be inscribed in a sphere of radius 3.

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**Question 5:** Can I write the data (known information) using the notation I introduced?  
Can I write the unknown using the notation I introduced?  
Can I state the problem using this notation?

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Which of these connections might I use to find the unknown?  
Can I use the notation I introduced to write an equation for this connection?  
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**Question 7:** Does my answer seem reasonable?  
What strategies did I use that might be helpful in another situation?

**PROBLEM:** Clint wants to build a dirt road from his ranch house to the highway so he can drive into town in the shortest time. The perpendicular distance from his ranch house to the highway is 4 miles. Town is 9 miles further down the straight highway. In good weather, Clint's average speed on the dirt road is 20 miles per hour. His average speed on the highway is 55 miles/hour. Where should Clint's dirt road join the highway? (Assume the land is flat.)

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**PROBLEM:** Two towers situated on level ground are 150 feet apart. One tower is 50 feet high and the other is 30 feet high. A guy wire is to run from the top of each tower to an anchor point between the two towers. Where should the anchor point be located so the total length of the guy wire is minimal?

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