Challenge:
Since many students find it helpful to have a solution guide to problems while studying, I challenge all MATH 160 students to create a solution guide to the problems on this worksheet. Send individual solutions to me, and I will make them available for all course sections to access by uploading them to the course webpage.

Instructions for Submissions:

- Write solutions on a clean sheet of paper, lined or unlined. Be sure to include the problem number. Typed solutions are appreciated, but not necessary.

- Submissions should be complete, legible, and contain good explanations. Explain everything as if you were teaching someone the material. Keep in mind that mathematicians love concise answers!

- Leave your signature or a sweet alias at the bottom of your solution so we know who gets credit for the solution. Fun pictures are optional but encouraged.

- Send all solutions to story@colostate.edu and brady.tyburski@colostate.edu or turn them in to your instructor in person.

Here's an example of what I'm looking for:

9. Suppose you own an apartment complex with 100 apartments. Every month, you make a total profit given by \( P = rq - 80(100 - q) - 18600 \), where \( r \) is the price you charge for rent, and \( q \) is the number of rooms rented. If the demand for rooms is given by \( q = 100 - \frac{r}{10} \), find the rent price and quantity of rooms rented that maximize your monthly profit, using calculus.

\[
\begin{align*}
\text{# 9. } P &= rq - 80(100-q) - 18600 \\
&= \frac{r^2}{10} - 8r - 18600
\end{align*}
\]

So \( P = \frac{r^2}{10} - 80 \left( 100 - \left( 100 - \frac{r}{10} \right) \right) - 18600 \)

\[
\begin{align*}
&= \frac{r^2}{10} - 8r - 18600 \\
&= \frac{-r^2}{10} + 92r - 18600
\end{align*}
\]

Then to find a max profit, we take the derivative and find the critical points:

\[
P' = \frac{-r}{5} + 92 \quad \text{(set to 0 or undefined)}
\]

\[
\text{\( r = \frac{4600}{9} \)}
\]

Now we need to show it's a max:

\[
P'' = -\frac{1}{5}, \quad \text{so} \quad P''(4600) = -\frac{1}{5}
\]

Since \( P''(4600) < 0 \), \( P \) is concave down at \( r = 4600 \) and it must be a maximum by 2nd derivative test.

We can only rent 100 rooms at the max, so we have an endpoint at \( q = 0, r = 4600 \) and \( q = 100, r = 0 \).

<table>
<thead>
<tr>
<th>( r )</th>
<th>( P(r) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>18600</td>
</tr>
<tr>
<td>4600</td>
<td>25000</td>
</tr>
<tr>
<td>100</td>
<td>20000</td>
</tr>
</tbody>
</table>

So the max profit is achieved when I rent \( q = 54 \) rooms at \( r = 4600 \) a piece.
Compute the following limits. (Please submit entire columns of solutions)

1. \( \lim_{x \to 8} \frac{x^2 - 6x - 16}{x^2 + x - 72} \)
2. \( \lim_{t \to 3} \frac{t - 3}{t^2 - 9} \)
3. \( \lim_{x \to 0} \frac{7x}{4x - 1} \)
4. \( \lim_{y \to 1} \frac{y^2 - 1}{2y + 2} \)
5. \( \lim_{x \to \infty} \frac{3x^2 + 2x + 1}{-9x^2 + 7x} \)
6. \( \lim_{x \to \infty} \frac{8x - 1}{7x^5 + 1} \)
7. \( \lim_{x \to \infty} (9x + 1) \)
8. \( \lim_{h \to 5} \frac{\sqrt{h} + 6 - 1}{h + 5} \)
9. \( \lim_{\theta \to 0} \frac{\sin \theta}{1 + \cos \theta} \)
10. \( \lim_{x \to \pi/2} \tan(x) \)
11. \( \lim_{x \to 0} \frac{\sin(x)}{x} \)
12. \( \lim_{x \to 0} \frac{1 - \cos(x)}{x} \)
13. \( \lim_{x \to 1} \frac{1}{x - 1} + \frac{1}{x^2 - 3x + 2} \)
14. \( \lim_{x \to 0} \cos(x + \sin(x)) \)
15. \( \lim_{x \to 4} \frac{(h - 1)^3 + 1}{h} \)
16. \( \lim_{x \to 1} \frac{x^2 + x - 2}{\tan(x) \sqrt{5 - x}} \)
17. \( \lim_{t \to 2} \frac{t^2 + 4t - 12}{t^2 - 2t} \)
18. \( \lim_{y \to -3} g(y) = \)
19. \( g(y) = \begin{cases} y^2 + 5 & y < -2 \\ 3 - 3y & y \geq -2 \end{cases} \)
20. \( \lim_{y \to -2} g(y) = \)
21. \( \frac{x^2 - 3x - 2}{x^2 - 2x + 1} \)
22. \( \frac{5x^2 - 5x - 5}{(x - 1)^3} \)
23. \( \frac{x^5 + 23x - 5}{\sqrt{x^2 + 1}} \)
24. \( \frac{3x - 5}{(x - 5)(x - 3)(x + 2)} \)
25. \( \frac{1}{1 - x} + \frac{1}{x + 1} \)
26. \( \frac{15\sqrt{x^2 - 3x - 2}}{\sqrt{x^2 - 2x + 1}} \)
27. \( \frac{1 + 2x + 5x^2 - 3x^3}{(1 + x)^3} \)
28. \( \frac{\sin(x)}{2x} \)
29. \( \frac{ax^5}{2(x + a)^5} \) (\(a\) is a constant)

In each case, find the values of A and B that make the following functions continuous.

30. \( f(x) = \begin{cases} Ax + B & x < -1 \\ 3 & x = -1 \\ Ax^2 - Bx + 2 & x > -1 \end{cases} \)
31. \( g(x) = \begin{cases} x^2 + x - 1 & x < -1 \\ \sin(\pi x) & -1 \leq x \leq 1 \\ Ax + B & x > 1 \end{cases} \)
Compute the following derivatives. (Please submit entire columns of solutions)

32. \( f(x) = 12x^2 + 2x + \sqrt{x} \)
33. \( y = 2\sqrt{3x^2} - \frac{3}{4x^2} - \pi^3 \)
34. \( g(x) = (3x + 4)^3 - 12x \)
35. \( f(x) = 8\sqrt{2x^2 - x} \)
36. \( y = (x^2 + 3x + 4)(3x + 2) \)
37. \( f(x) = \frac{1 - x - 4\sqrt{x}}{x} \)
38. \( g(t) = \frac{4 - 3t}{(2t - 3)^2} \)
39. \( \frac{d}{dx}(\sin x) = \)
40. \( \frac{d}{dx}(\cos x) = \)
41. \( \frac{d}{dx}(\tan x) = \)
42. \( \frac{d}{dx}(\cot x) = \)
43. \( \frac{d}{dx}(\sec x) = \)
44. \( \frac{d}{dx}(\csc x) = \)
45. \( r(y) = \sin(y) \tan(y) \)
46. \( h(x) = \tan(4 - x) + 2\sin(\pi x) \)
47. \( g(t) = \frac{2t - 5}{t^2 + 3\sec(t)} \)
48. \( f(x) = 2x + e^\pi^5 - x^3 - \frac{1}{\sqrt{x}} \)
49. \( x(t) = \frac{4 - t}{t + 2} \)
50. \( v(x) = \frac{7}{2 - x} - \frac{3}{4x^3} \)
51. \( f(s) = \sqrt[3]{s}(7s^2 - \pi s) \)
52. \( h(x) = \frac{3x - x^{10} + 7x - 1}{3 - \sqrt{x}} \)
53. \( y = (x^5 + x^4 - x^3 - x^2 + x + 1)(\sqrt{x} - \frac{3}{\sqrt{x}} + \frac{4}{\sqrt{x}}) \)
54. \( f(x) = ax^2 + \frac{b}{3}x^3 + c \)  \( a, b, c \) are constants
55. \( s(z) = |z - 3| \)
56. \( m(t) = \sqrt{t^5 + \frac{4}{\sqrt{t^3}}} + 15 \)
57. \( g(x) = \sum_{k=1}^{10} \frac{x^k}{k!} \)
58. \( h(x) = \cos(2x) + i\sin(2x) \)
59. \( y(t) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x - a)^n \)

60. For the functions \( h(x) = 2|x - 2| \) and \( p(x) = \sqrt{|x|} \), find \( h'(2) \) and \( p'(0) \) from the definition (since that is your only option). If these are not differentiable, explain why and how they are different.

61. Let \( g(x) = \begin{cases} \sqrt{1 - x} & x \leq 1 \\ -4x + 4 & x > 1 \end{cases} \)

Is \( g(x) \) differentiable? If not, explain why using the limit definition of the derivative.

In each case, find the values of \( A \) and \( B \) that make the function differentiable for all \( x \) in the domain.

62. \( h(x) = \begin{cases} A\sin(\frac{\pi x}{2}) & x < 1 \\ 3 & x = 1 \\ Bx + A & x > 1 \end{cases} \)
(a) Find the derivative of \( g(t) \).
(b) Use the information provided in the table below to find the value of \( g'(\pi) \).

<table>
<thead>
<tr>
<th>( t )</th>
<th>( f(t) )</th>
<th>( f'(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t = 0 )</td>
<td>2</td>
<td>-\frac{1}{2}</td>
</tr>
<tr>
<td>( t = \pi )</td>
<td>5</td>
<td>-2</td>
</tr>
</tbody>
</table>

63. \( q(x) = \begin{cases} x^2 + Ax + B & x < 1 \\ Bx & x \geq 1 \end{cases} \)

64. Use \( g(t) = f(\tan(t)) \) to answer the following.
Compute the following antiderivatives. (Please submit entire columns of solutions)

65. \( \int x\sqrt{6-x^2} \, dx \)
66. \( \int \cos(t) \sqrt{2 + \sin(t)} \, dt \)
67. \( \int \frac{7x^4}{(3 + x^5)^6} \, dx \)
68. \( \int 2y(y - 4)^3 \, dy \)
69. \( \int 10x^4\sqrt{2x^5} - 7 \, dx \)
70. \( \int 2\omega(\omega^2 - 1)^7 \, d\omega \)
71. \( \int \frac{2x^5}{(\frac{1}{3}x^6 - 1)^2} \, dx \)
72. \( \int 3y\sqrt{7 - 3y^2} \, dy \)
73. \( \int \frac{1}{\sqrt{5s + 4}} \, ds \)
74. \( \int \tan^2(x) \sec^2(x) \, dx \)
75. \( \int \frac{1}{x^2} \sqrt{2 - \frac{1}{x}} \, dx \)

Compute the following definite integrals. (Please submit entire columns of solutions)

76. \( \int_0^\pi \cos^2(p) \sin(p) \, dp \)
77. \( \int_{-2}^{-1} \frac{x}{(x^2 + 2)^2} \, dx \)
78. \( \int_1^3 \frac{t + 2}{\sqrt{t^2 + 4t + 4}} \, dt \)
79. \( \int_{\sqrt{\pi}/2}^{\pi} \theta \sin(\theta^2) \, d\theta \)
80. \( \int_{-1}^{0} \sqrt{y + 1} \, dy = \)
81. \( \int_{-\pi/4}^{0} \tan(x) \sec^2(x) \, dx = \)
82. \( \int_{-1}^{1} t^3(1 + t^4) \, dt = \)
83. \( \int_0^{1} \frac{5r}{(4 + r^2)^2} \, dr = \)
84. \( \int_{-1}^{0} \frac{4x}{x^2 + 1} \, dx = \)
85. \( \int_{0}^{\pi/4} \tan^4(x) \sec^2(x) \, dx \)
86. \( \int_{1}^{7} \sqrt{\omega^2 + 1} \, d\omega \)
87. \( \int_{0}^{1} \frac{x^3}{\sqrt{x^4 + 9}} \, dx \)
88. \( \int_{1}^{9} t\sqrt{4 + 5t} \, dt \)
89. \( \int_{0}^{\pi/2} (1 - \cos(3x)) \sin(3x) \, dx \)

Decide whether the following statements are correct.

90. \( \int \sqrt{2x + 1} \, dx = \sqrt{x^2 + x} + C \)
91. \( \int \frac{x^2 + x + 2}{x^3} \, dx = \frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x + C \)

Find the areas of the following regions. Include sketches of the regions.

92. The total area of the regions bounded by \( f(x) = x^3 \) and \( g(x) = x \).
93. The region bounded by \( x = 12y^2 - 12y^3 \) and \( x = 2y^2 - 2y \), with \( y \geq 0 \).
94. The region bounded by \( y = x \) and \( y = \frac{x^4}{4} \) with \( y \leq 1 \).
State whether the following statements are true or false. If a statement is true, explain why it is true. If a statement is false, provide a counterexample to the statement.

95. If \( f'(a) \) exists, then \( f(x) \) is continuous at \( x = a \).

96. If \( \lim_{x \to 2} f(x) = f(2) \), then \( \lim_{h \to 0} \frac{f(2 + h) - f(2)}{h} \) exists.

97. If the velocity of a particle is increasing, its position must be positive.

98. If \( f'(x) = g'(x) \), then \( f(x) = g(x) \).

99. If \( f(x) \) is increasing, then \( \int_{-2}^{2} f(x) \, dx > 0 \).

100. If \( \int_{a}^{b} g(x) \, dx = 0 \), then \( g(x) = 0 \) on \([a, b]\).

101. If \( f(x) > 0 \), then \( \int_{a}^{b} f(x) \, dx > 0 \) for all \( a \) and \( b \).

102. If \( f(x) \) is not continuous at \( x = 5 \), then \( \int_{1}^{6} f(x) \, dx \) does not exist.

103. If \( f(x) \) and \( g(x) \) are differentiable, then the total area bounded by \( f \) and \( g \) on the interval \([0, 4]\) is given by \( \int_{0}^{4} [f(x) - g(x)] \, dx \).

104. If \( h'(4) = 0 \), then \( h \) must have a maximum or minimum at \( x = 4 \).

105. If \( p(x) \) and \( q(x) \) are both increasing functions, then \( f(x) = p(x) + q(x) \) must be increasing.

106. \( f(x) = |x| \) has infinitely many tangent lines at \( x = 0 \).

107. If the derivative does not exist at a point, then a tangent line cannot exist at that point.

108. \( f(x) \) defined on \([0, 6]\), then it must attain both an absolute maximum and an absolute minimum on \([0, 6]\).

109. \( f(x) = |x + 5| \) is not differentiable at every \( x \) in its domain.

110. If \( g(x) \) is continuous on the union of intervals \([2, 4] \cup [5, 7]\), then \( g(x) \) must attain an absolute maximum on \([2, 4] \cup [5, 7]\).

111. If \( h(x) \) has a cusp at \( x = 15 \), then \( h(x) \) has a vertical asymptote at \( x = 15 \).

112. Piecewise functions are not differentiable.

113. Given that \( f(1) = f(3) = 0 \) and \( f'(2) = 0 \), then \( f(x) \) must be continuous on the interval \([1, 3]\).

114. A curve that doesn’t pass the vertical line test at \( x = -1 \) cannot have tangent lines for points where \( x = -1 \).

115. If \( \lim_{h \to 0} \frac{f(2 + h) - f(2)}{h} = 4 \), then \( f(2) = \lim_{x \to 2} f(x) \).

116. If \( f(3) = 2 \) and \( \lim_{x \to 3} f(x) = 2 \), then \( \lim_{x \to 3} f(x) = 2 \).

117. If \( f(x) \) has a vertical asymptote at \( x = -2 \), then \( f(-2) \) is undefined.

118. If \( g(x) \) is not continuous, then \( g(x) \) is not differentiable.
119. If \( \lim_{x \to \infty} p(x) = 3 \), then \( \lim_{x \to -\infty} p(x) = 3 \)

120. A midpoint sum with any number of partitions will find the exact area under \( f(x) = x + 5 \) on any interval \([a, b], a < b\).

121. Midpoint sums are always more accurate than left or right hand sums.

122. If \( g''(c) = 0 \), then there is an inflection point at \( x = c \).

123. If \( \lim_{x \to 17} g(x) \) does not exist, then \( g(17) \) does not exist.

124. If \( q'(r) < 0 \) for \( r < 1 \) and \( q'(r) > 0 \) for \( r > 1 \), then \( q(r) \) must have an absolute minimum at \( r = 1 \).

125. If a woman travels 200 mi in 2.5 hrs, she must have exceeded the 60 mph speed limit at some point during her drive.

126. It’s impossible for a function to have an absolute max on on open interval.

127. Let \( f'(x) > 0 \) for all \( x \). Then \( g(x) = f(f(x)) \) is always increasing.

128. If \( f(x) \) is increasing on \([a, b]\), then \( \int_a^b f'(x)dx > 0 \).

129. If a certain type of glass is cooled faster than at a rate of 45\(^\circ\)F per second, it will shatter. Suppose the dining hall just put out some freshly-washed glasses at 150\(^\circ\)F, and your friend grabs one and fills it with ice cold water. If the glass cools to 50\(^\circ\)F after two seconds, we know for sure that the glass shattered.

130. Suppose we have the same type of glass, and that you’ve been having some trouble with your hot water heater. You fill your dishwasher with glasses at room temperature (70\(^\circ\)) and run it, and the glass comes out again at 120\(^\circ\) 30 minutes later. We are guaranteed that none of the glasses shattered due to temperature changes.

131. If \( f(x) \) is differentiable on \([2, 5]\), then it must have both a maximum value and minimum value on \([2, 5]\).

132. If \( f(a) = f(b) = 0 \), then there must be some point \( c \) in \([a, b]\) at which \( f'(c) = 0 \).

133. If \( f(x) \) is continuous and \( f(a) = f(b) = 5 \), then there must be some point \( c \) in \([a, b]\) at which \( f'(c) = 0 \).

134. *Challenge* If two racers start a race at the same time and end at the same time, it is possible that they were never moving at exactly the same speed over the course of the race.

135. *Challenge* If \( f(x) \) has jump discontinuities at every integer, then the integral \( \int_0^9 f(x)dx \) is undefined.

136. *Challenge* \( \lim_{b \to \infty} \int_1^b \frac{1}{x^p}dx \) is undefined for every number \( p \).

137. *Extreme Challenge* If \( h(x) \) is differentiable, then \( h'(x) \) is continuous.

138. *Extreme Challenge* \( \int_{-1}^1 \frac{1}{x}dx = 0 \)
139. *Challenge* In a beehive, each cell is a hexagonal prism, open at one end with a trihedral angle at the other, as in the figure. It is believed that bees form their cells in such a way as to minimize the amount of wax needed to construct the hive. Based on the geometry of the cell, we know that each cell has a surface area of

\[ S = 6sh - \frac{3}{2} s^2 \cot \theta + 3s^2 \frac{\sqrt{3}}{2} \csc \theta \]

Find the angle that minimizes the surface area of the cell (suppose \( s \) and \( h \) are given). Actual measurements of this angle show that bees seldom differ from the optimal angle by more than 2 degrees!

140. Find the maximum product of two non-negative numbers such that three times the first number plus the second is 100.

141. The rate (in \( mg \) of carbon per cubic meter per hour) at which photosynthesis takes place for a species of phytoplankton is modeled by \( R = \frac{100I}{I^2 + I + 4} \), where \( I \) is the light intensity (in thousands of foot-candles). What level of light intensity maximizes \( R \), the rate of photosynthesis?

142. A farmer wants to fence an area of 1.5 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides. What dimensions should he use to minimize the amount of fence he needs?

143. A box with a square base and no top must have a volume of 32,000 \( cm^3 \). Find the dimensions of the box that minimize the amount of material needed.

144. A piece of wire 10 m long is cut into two pieces. One piece is bent into a square, and the other is bent into an equilateral triangle. How should the wire be cut so that the area enclosed by the two shapes is a maximum?

145. A fence 8 ft tall runs parallel to a tall building at a distance of 4 ft from the building. What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building?

146. A baseball team plays in a stadium that holds 55,000 spectators. With ticket prices at \$10, the average attendance is 27,000. When ticket prices dropped by \$2, attendance increased to 33,000. Assuming this pattern holds for any price change, what price should the stadium charge to maximize its revenue?

147. Your company produces bouillon cubes. The bouillon cubes are specified to have a volume of 3375 mm\(^3\).

(a) What must the side length measure to result in a volume of 3375 mm\(^3\)?
(b) Suppose that the Packaging department says that the volume cannot exceed $3405 \text{mm}^3$ and the company’s Chief Financial Officer says it cannot be less than $3345 \text{mm}^3$. What values must the side lengths be to ensure both restrictions?

(c) Sketch the graph of volume $V(x)$ as a function of side length $x$. Then sketch the interval on the $V$-axis that must contain the cubes volumes. Next, sketch the interval of $x$-values which ensures that $V(x)$ falls in the interval on the $V$ axis.

148. The physicists at CSU are building a top-secret machine with a capacitor that, when charged, needs to hold 9 GJ (gigajoules) of energy. If this stored energy varies by more than $\pm \frac{1}{2}$ GJ, then a black hole will form and destroy the earth. The energy stored by this capacitor is given by 

$$U(v) = \frac{2.55v^2}{2},$$

where $v$ is the applied voltage and $U$ is the stored energy.

(a) Find $v > 0$ such that $U(v) = 9 \text{ GJ}$. We will call this value $v_0$. Round to at least 4 decimal places.

(b) What interval around $v_0$ will ensure that corresponding output values are close enough to $U = 9 \text{ GJ}$?

(c) How close to $v_0$ do we need to keep $v$ to ensure that $U$ remains close enough to 9 GJ so that the earth is not doomed? Round to at least 4 decimal places. Explain using words why you chose this value.

(d) What is the maximum amount of error that can occur on either side of $v_0$.

149. Can the MVT be applied to $f(x) = x^3 + 2x^2 - x$ on $[-1, 2]$? If so, determine all the numbers $c$ which satisfy the conclusions of the Mean Value Theorem.

150. Marty McFly rides Dr.Brown’s time-travelling Delorean to class, because he’s able to teleport the three miles in just two seconds. What’s Marty’s average velocity on the way to class? Can we use the mean value theorem to show that Marty must have been travelling that velocity at some point in his trip? (Hint: Think about what Marty’s position function would look like, and then check the assumptions of the MVT)

151. Suppose you know the function $f(x)$ is differentiable on $[-10, 10]$, and that it has a local maximum at $x = -3$ so that $f(-3) = M$ and a local minimum at $x = 7$ so that $f(7) = m$. Which of the following are true? Explain why each statement is true, or give a counterexample.

(a) There must be some $c \in (-3, 7)$ so that $f'(c) = \frac{m-M}{10}$.

(b) There must be some $c \in (-3, 7)$ at which $f(c) = \frac{m+M}{2}$.

(c) $f(x)$ must achieve its absolute max somewhere on the interval $[-3, 7]$.

(d) $f(x)$ has both a maximum and a minimum value on the interval $[-3, 7]$.

(e) We know that $m < M$.

152. Suppose we are tracking the pace of a short distance runner. We measured the speed, $(v(t))$, of the runner at 1 minute intervals for the duration of his training run. (We gave him a headstart to get up to speed before time 0).

<table>
<thead>
<tr>
<th>Time (mins)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed (mph)</td>
<td>14</td>
<td>13</td>
<td>12</td>
<td>11</td>
<td>11</td>
<td>14</td>
<td>16</td>
</tr>
</tbody>
</table>

(a) Create an upper estimate for the distance he ran over those 6 minutes.

(b) Create a lower estimate for the distance he ran over the 6 minute session.
153. Oil is leaking out of a tanker damaged at sea. The rate that oil is leaking out of the tanker is recorded in the table below:

<table>
<thead>
<tr>
<th>t (hours)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>a (gal/hr)</td>
<td>50</td>
<td>70</td>
<td>92</td>
<td>136</td>
<td>130</td>
<td>240</td>
<td>369</td>
<td>516</td>
<td>720</td>
</tr>
</tbody>
</table>

(a) Find an upper estimate and lower estimate for the total quantity of oil that has escaped after 5 hours.

(b) Repeat part (a) for the quantity of oil that has escaped after 8 hours.

154. Estimate the area enclosed by \( f(x) = x^2 - 1 \) and \( g(x) = 1 - x^2 \) using a left endpoint sum with 8 partitions. Sigma notation is not necessary, but sketch a picture of the rectangles used in the sum.

155. *Challenge* If \( \int_{2}^{7} f(x)dx = -9 \), \( \int_{-2}^{-4} f(x)dx = 2 \), and \( f(x) \) is even, what is \( \int_{4}^{7} f(x)dx \)?

156. Consider the function \( g(x) \) is defined by \( g(x) = \int_{0}^{x} f(t)dt \) and the graph of \( f \) is given below.

(a) Find \( g(2) \)  
(b) Find \( g(-2) \)  
(c) Find \( g(-5) \)  
(d) Find \( g(5) - g(4) \)  
(e) Find \( g'(-1) \)  
(f) Find \( f(2) \)  
(g) Find \( g''(2) \)  
(h) Find \( g''(5) \)

157. Evaluate the following expressions.

(a) \( \frac{d}{dx} \int_{1}^{x^2} \sin(t)dt \)  
(b) \( \frac{d}{dx} \int_{-3x}^{2} t^2 + t dt \)  
(c) \( \frac{d}{dx} \int_{-x}^{x} t^3 dt \)  
(d) \( \frac{d}{dx} \int_{a}^{\cos(x)} |t| dt \)
158. Let \( F(x) = \int_0^x f(t) dt \) where \( f(x) \) is the function whose graph is shown below.

(a) Where are the critical points of \( F(x) \)?
(b) Does \( F(x) \) have any local maxima? Where are they? Explain how you know they are maxes. Don’t forget about the endpoints!
(c) Does \( F(x) \) have any local minima? Where are they? Explain how you know they are maxes. Don’t forget about the endpoints!
(d) Assuming the pattern started in the graph holds, will \( F(x) \) ever be negative for any \( x > 0 \)?

159. The graph below is the equation of \((x^2 + y^2 - 8x)^2 = 4(x^2 + y^2)\). Answer the following questions that pertain to it.

(a) Verify that the point \((3, 3\sqrt{3})\) is on the curve.
(b) Find an expression for \( \frac{dy}{dx} \).
(c) What is the equation of the tangent line to the curve at the point \((3, 3\sqrt{3})\)?

160. Find \( \frac{dy}{dx} \) for the following. (Please submit entire columns of solutions)

\[
\begin{align*}
\text{(a)} & \quad 3y^3 + x^2 = 5 & \quad \text{(e)} & \quad \sqrt{xy} = x + 3y \\
\text{(b)} & \quad y^4 - y = x^3 + x & \quad \text{(f)} & \quad 4\sin(2y) \cos(x) = 2 \\
\text{(c)} & \quad x^2y + y^4 - 3x = 8 & \quad \text{(g)} & \quad x \sec(y) = xy \\
\text{(d)} & \quad x^2y + 2xy^2 = x + y & \quad \text{(h)} & \quad xy^2 - \tan(\pi xy) = 2x
\end{align*}
\]
161. Sketch a graph of a function with the following properties:

<table>
<thead>
<tr>
<th>Condition</th>
<th>$f(x)$</th>
<th>$f'(x)$</th>
<th>$f''(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x &lt; -4$</td>
<td>$\lim_{x \to -\infty} f(x) = -\infty$</td>
<td>$f'(x) &gt; 0$</td>
<td>$f''(x) &lt; 0$</td>
</tr>
<tr>
<td>$x = -4$</td>
<td>$f'(-4) = 0$</td>
<td>$f''(-4) &lt; 0$</td>
<td></td>
</tr>
<tr>
<td>$-4 &lt; x &lt; -1$</td>
<td>$\lim_{x \to -1^+} f(x) = -2$</td>
<td>$f'(x) &lt; 0$</td>
<td>$f''(x) &lt; 0$</td>
</tr>
<tr>
<td>$x = -1$</td>
<td>$f(-1) = -3$</td>
<td>$f''(-1) \mathrm{DNE}$</td>
<td></td>
</tr>
<tr>
<td>$-1 &lt; x &lt; 1$</td>
<td>$\lim_{x \to 1^-} f(x) = \infty$</td>
<td>$f' &lt; 0$</td>
<td>$f'' &gt; 0$</td>
</tr>
<tr>
<td>$x = 1$</td>
<td>$f(1) = 0$</td>
<td>$f''(1) \mathrm{DNE}$</td>
<td></td>
</tr>
<tr>
<td>$1 &lt; x &lt; 3$</td>
<td>$f'(x) &gt; 0$</td>
<td>$f''(x) &lt; 0$</td>
<td></td>
</tr>
<tr>
<td>$x = 3$</td>
<td>$f'(3) = 0$</td>
<td>$f''(3) \mathrm{DNE}$</td>
<td></td>
</tr>
<tr>
<td>$x &gt; 3$</td>
<td>$\lim_{x \to \infty} f(x) = 0$</td>
<td>$f'(x) &lt; 0$</td>
<td></td>
</tr>
</tbody>
</table>

162. Identify which graph is $f(x)$, which graph is $f'(x)$, and which graph is $f''(x)$. Explain how you know.

![Graph Image]

163. There are three functions plotted in the coordinate system below corresponding to the position, velocity, and acceleration of a particle. Determine which label goes with each graph, and explain your answer.

![Graph Image]
164. Suppose that we have a function, \( f(t) \). Below is the graph of the derivative of \( f(t) \) on the interval \([0, 1]\). Be sure to consider the endpoints in your answers.

(a) Identify the critical point(s) of \( f(t) \).
(b) Identify the interval(s) on which \( f(t) \) is increasing.
(c) Identify the interval(s) on which \( f(t) \) is decreasing.
(d) Identify the interval(s) on which \( f(t) \) is concave up.
(e) Identify the interval(s) on which \( f(t) \) is concave down.
(f) Identify where \( f(t) \) has local maxima, if applicable.
(g) Identify where \( f(t) \) has local minima, if applicable.

165. Let \( f''(x) = 18x + 6 \) and suppose \( f(2) = 4 \) and that \( f \) has a horizontal tangent line at \( x = 3 \). Find \( f(x) \).

166. A stone was dropped off a cliff and hit the ground with a speed of 120 \( \frac{m}{s} \). What is the height of the cliff?

167. A car is traveling 50 \( \frac{mi}{hr} \) when the brakes are applied, producing a constant deceleration of 22 \( \frac{ft}{s} \). How far does the car travel after the brakes are applied?

168. A company estimates that the marginal cost (in dollars per item) of producing \( x \) items is \( 1.92 - .0002x \). If the cost of producing one item is \$562, find the cost of producing 100 items.

Find the volume of the regions described below.

169. (Textbook problem 10, page 321) The base of the solid is the unit disc \( (x^2 + y^2 \leq 1) \), and the cross sections are isosceles right triangles with one leg in the disc. (picture in book)

170. (Textbook problem 7, page 321) The base of the solid is the region bounded by \( y = 3x, y = 6, \) and \( x = 0 \). The cross sections are rectangles of height 10.

171. Find the volume of the solid generated by revolving the region bounded by \( y = 4 - x^2 \) and \( y = 2 - x \) around the x-axis.

172. Find the volume of the solid generated by revolving the region bounded by \( x = y - y^2 \) and \( x = 0 \) around the y-axis.

173. Find the volume of the solid generated by revolving the region bounded by \( y = x^2 \) and \( x = y^2 \) about \( x = -1 \).
174. The graph of $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$ is called an astroid. Find the arc length of the astroid by first finding the arc length of a segment of the curve $y = (1 - x^{\frac{2}{3}})^{\frac{3}{2}}$ on $\frac{\sqrt{2}}{4} \leq x \leq 1$. Draw a picture and explain what the arc length of the whole astroid will be.

175. Find the arc length of the curve $y = 1 + x^{\frac{2}{3}}$ on $0 \leq x \leq 1$.

Don’t forget that you can go to any MATH 160 professor’s help hours. We are all happy to help!