2. Let \( r(x) = (F \circ G)(x) \) and \( t(x) = (G \circ F)(x) \).

Use the graphs of the functions \( G(x) \) and \( F(x) \) below to find the following derivatives. You may need to estimate the slope of graphs at particular points. If an answer is undefined, state why in 1-2 sentences.

\[
(a) \quad r'(3) = \frac{d}{dx} (F \circ G)(x) \bigg|_{x=3} = F'(G(3)) \cdot G'(3)
\]

\[
= F'(6) \cdot 2 = \frac{-4(2)}{2} = -8
\]

\[
\text{from graph}
\]

\[
(b) \quad t'(2) = \frac{d}{dx} (G \circ F)(x) \bigg|_{x=2} = G'(F(2)) \cdot F'(2)
\]

\[
= G'(0) \cdot \frac{-1}{2} = \frac{-1}{2}(4) = -2
\]

\[
\text{from graph}
\]
3. Calculate the indicated derivatives by using the Differentiation Rules (Theorems). Answers must be accompanied by supporting work that shows how you calculated the derivative. **You do not need to simplify your answers on these problems.** If you do simplify an answer, you must simplify correctly. Answers will be scored right or wrong. Don’t expect partial credit for incorrect answers or answers with no supporting work.

(a) \( g(x) = \frac{\sqrt{x}}{\tan(3x)} \)

\[ g'(x) = \frac{\tan(3x)\sqrt{1x} - \sqrt{1x} \cdot 3\sec^2(3x)}{\tan^2(3x)} \]

(or can use product rule)

\[ g(x) = \sqrt{1x} \cdot \cot(3x) = \sqrt{1x} \cdot \cot(3x) + \sqrt{1x} \cdot (-\csc^2(3x)3) \]

(b) \( h(r) = \sqrt[3]{2r^3 - 5r + 1} \)

\[ h'(r) = \frac{1}{3}(6r^2 - 5)(2r^3 - 5r + 1)^{-2/3} \]

(c) \( f(x) = \sin\left(\frac{x}{2}\right) \cdot \cos\left(\frac{2}{x}\right) = \sin\left(\frac{1}{x}\right) \cos\left(2x^{-1}\right) \)

\[ \left(\frac{1}{2} \cos\left(\frac{x}{2}\right)\right)\cos\left(\frac{2}{x}\right) + \sin\left(\frac{1}{x}\right)(-2x^{-2})(-\sin\left(2x^{-1}\right)) \]

Equivalent form:

\[ \frac{1}{2}\cos\left(\frac{x}{2}\right)\cos\left(\frac{2}{x}\right) + \frac{2}{x^2}\sin\left(\frac{x}{2}\right)\sin\left(\frac{2}{x}\right) \]
5. Use \( f(x) = |x + 1|, g(x) = (x + 1)^{1/3}, \) and \( k(x) = (x + 1)^{2/3} \) to answer the following:

(a) \( f(x) \) is not (circle one) continuous at \( x = -1 \).

(b) \( g(x) \) is not (circle one) continuous at \( x = -1 \).

(c) \( k(x) \) is not (circle one) continuous at \( x = -1 \).

(d) Although the functions \( f(x), g(x), \) and \( k(x) \) are not differentiable at \( x = -1 \), the slopes of the curves each behave differently around \( x = -1 \).

**Description of \( f(x) \):**

\[
\lim_{h \to 0^-} \frac{f(-1+h) - f(-1)}{h} = -1
\]
So the slopes on the left side of \( x = -1 \) are:
(provide a brief description about the slope of \( f(x) \) on the left side of \( x = -1 \))

**Constant value of \(-1\)**

\[
\lim_{h \to 0^+} \frac{f(-1+h) - f(-1)}{h} = \infty
\]
So the slopes on the left side of \( x = -1 \) are:
(provide a brief description about the slope of \( f(x) \) on the right side of \( x = -1 \))

**Description of \( g(x) \):**

\[
\lim_{h \to 0^-} \frac{g(-1+h) - g(-1)}{h} = \infty
\]
So the slopes on the left side of \( x = -1 \) are:
(provide a brief description about the slope of \( g(x) \) on the left side of \( x = -1 \))

**Increasing w/out bound**

\[
\lim_{h \to 0^+} \frac{g(-1+h) - g(-1)}{h} = \infty
\]
So the slopes on the right side of \( x = -1 \) are:
(provide a brief description about the slope of \( g(x) \) on the right side of \( x = -1 \))

**Increasing w/out bound**
Problem 5 continued...

**Description of k(x):**

\[
\lim_{{h \to 0^-}} \frac{k(-1 + h) - k(-1)}{h} = -\infty
\]

So the slopes on the left side of \( x = -1 \) are:

(provide a brief description about the slope of \( k(x) \) on the left side of \( x = -1 \))

\[ \text{decreasing w/ out bound} \]

\[
\lim_{{h \to 0^+}} \frac{k(-1 + h) - k(-1)}{h} = \infty
\]

So the slopes on the right side of \( x = -1 \) are:

(provide a brief description about the slope of \( k(x) \) on the right side of \( x = -1 \))

\[ \text{increasing w/ out bound} \]

(e) Use the results of your discussion in (d) to come up with a simple and concise way to clearly distinguish between a corner, cusp, and a vertical tangent. i.e. How can one determine, in general, if a function has a corner, cusp, or vertical tangent?

→ The slopes on either side of a cusp approach \( +\infty \) & \( -\infty \).

→ The slopes on either side of a corner are unequal finite numbers.

→ The slopes on either side of a vertical tangent are both \( \infty \) or both \( -\infty \).