Written HW 03 - Due Tuesday, February 6

Name: ________________________________

1. Use $g(x) = \begin{cases} \cos\left(\frac{\pi x}{2}\right) & \text{if } x < 4 \\ \frac{x}{2} & \text{if } x = 4 \\ -(x - 5)^2 + 2 & \text{if } x > 4 \end{cases}$ to answer the following.

Be sure to show ALL WORK AND STEPS. When evaluating parts (a), (b), and (c) below, be sure to be clear about which piece of the function you are using. For example, if you are using $-(x - 5)^2 + 2$, then state that you are using piece 3.

(a) Determine the value of $g(4)$:

If $x = 4$, then use second piece: $g(4) = \frac{4}{2} = 2$

(b) Determine the value of $\lim_{{x \to 4^-}} g(x)$:

The left side of $x = 4$ means need to use first piece

$\lim_{{x \to 4^-}} g(x) = \lim_{{x \to 4^-}} \cos\left(\frac{\pi x}{2}\right) = \cos\left(\frac{4\pi}{2}\right) = \cos(2\pi) = 1$

(c) Determine the value of $\lim_{{x \to 4^+}} g(x)$:

The right side of 4 means need to use third piece:

$\lim_{{x \to 4^+}} g(x) = \lim_{{x \to 4^+}} -(x - 5)^2 + 2 = -(4 - 5)^2 + 2 = -(-1)^2 + 2 = 1 + 2 = 3$
2. Use \( f(x) = \begin{cases} \frac{1}{2}x^2 + x + a & x < -1 \\ \frac{a}{x} + \frac{b}{2} & -1 \leq x < 1 \\ \frac{a}{x} + \frac{b}{2} & x \geq 1 \end{cases} \) to answer the following:

Find the value(s) of \( a \) and \( b \) that make \( f \) continuous everywhere. Your work should include limit calculations. If your work does not include limit calculations, points will be taken off on this problem. Explain how you know that the value(s) you found make \( f \) continuous everywhere.

Notes: Each piece is continuous on its interval. The first two pieces are polynomials, which are continuous. The third piece would only have an issue at \( x=0 \), which is not part of \( x \geq 1 \). So need to insure that the functions is continuous where the pieces change at \( x=-1, 1 \).

Thus need \( \lim_{x \to -1} f(x) = f(-1) \) and \( \lim_{x \to 1} f(x) = f(1) \)

\[
\begin{align*}
\lim_{x \to -1} f(x) &= \lim_{x \to -1} \left( \frac{1}{2}x^2 + x + a \right) = \frac{1}{2} - 1 + a \\
\lim_{x \to 1} f(x) &= \lim_{x \to 1} \left( \frac{a}{x} + \frac{b}{2} \right) = -b + \frac{a}{2}
\end{align*}
\]

\[
\begin{align*}
-\frac{1}{2} + a &= -b + \frac{a}{2} \\
\frac{a}{2} &= b + 5
\end{align*}
\]

\[
\begin{align*}
a - \frac{1}{2} &= -b + 5 \\
2a + \frac{b}{2} &= 10
\end{align*}
\]

\[
\begin{align*}
a - \frac{3}{2} &= b + 5 \\
2a + 4 &= 10 \\
2a &= 6 \\
\Rightarrow a &= 3
\end{align*}
\]

\[
\begin{align*}
y = 5 \\
\frac{b}{2} &= 6 \\
b &= 3 + \frac{3}{2} \\
&= 5
\end{align*}
\]

(d) What can be said about the absolute extrema of \( g \) if we consider it on its entire domain rather than \([-2, \frac{1}{2}]\)? Provide a justification for your answer.

\( g(x) \) is continuous on its domain which is \((-\infty, 1) \cup (1, \infty)\).

However, this is not a closed interval, so we cannot apply the EVT.

5. Indicate whether each of the following statements is True or False. If the statement is true, explain how you know it’s true. If it is false, give a counterexample and explain why it is a counterexample. (A counterexample is an example that shows the statement is false.)

(a) Every rational function has at least one vertical asymptote. \[\text{False}\]

\[
\begin{align*}
f(x) &= \frac{x}{x+1} \\
\text{No such } x \text{-value such that } \lim_{x \to -1} f(x) = \pm \infty \text{ or } \lim_{x \to 1} f(x) = \pm \infty \\\nf(x) &= \frac{x+1}{x+1} \\
\lim_{x \to -1} f(x) &= 1 \text{ (not } \pm \infty) \\
f(x) &= \text{polynomial, } f(x) = \text{constant}
\end{align*}
\]
5(c) If \( f(x) \) is defined on \([-3, 4]\), then \( f \) must attain an absolute maximum on \([-3, 4]\). **False**

In order to apply the EVT, \( f(x) \) needs to be continuous. Being defined is not sufficient.

\( f(x) \) is defined on \([-3, 4]\), but does not attain an abs. max.