1. Let \( g(x) = \begin{cases} 
2x|x-1| & \text{if } x < 1 \\
\frac{x-1}{|x-1|} & \text{if } x > 1 \\
\frac{2x|x-1|}{x-1} & \text{if } x = 1
\end{cases} \)

(a) Find \( \lim_{x \to 1^+} g(x) \)

(b) Find \( \lim_{x \to 1^-} g(x) \)

(c) Does \( \lim_{x \to 1} g(x) \) exist? Explain why or why not in 1-2 sentences.

(d) Sketch the graph of \( g \).
2. Each of the following statements are sometimes true and sometimes false. Draw a graph of a function for both cases. Provide a 1-2 sentence explanation of why your graph satisfies the true statement. Provide a 1-2 sentence explanation of why your graph satisfies the false statement.

(a) If \( \lim_{x \to -2} f(x) \) exists, then \( \lim_{x \to -2} f(x) = f(-2) \)

\[ \text{Graph for when statement is true: } \]
\[ \text{Graph for when statement is false: } \]

Explanation:  

(b) If \( f(\pi) \) exists, then \( \lim_{x \to \pi} f(x) \) must exist.

\[ \text{Graph for when statement is true: } \]
\[ \text{Graph for when statement is false: } \]

Explanation:  

3. Determine whether the statement is true or false. If it is true, explain why. If it is false, explain why or give a counterexample. (A counterexample is an example of a function for which the “if” part of the statement is true, but the “then” part is false.) A graph with an explanation in words can be used as a counterexample.

(a) If \( \lim_{x \to 0} f(x) = 0 \) and \( \lim_{x \to 0} g(x) = 0 \), then \( \lim_{x \to 0} \frac{f(x)}{g(x)} \) does not exist.

(b) If \( f(3) = 2 \) and \( \lim_{x \to 3^+} f(x) = 2 \), then \( \lim_{x \to 3^-} f(x) = 2 \).

(c) If \( h(0) \) does not exist, then \( \lim_{t \to 0} h(t) \) cannot exist.
4. Scotty needs to evaluate the limit of an oscillating function. His work is below.

(a) Evaluate Scotty’s work. If errors occur from one step to the next, state the errors on the line provided. If there is no error in the step, draw a smiley face.

\[
(1) \lim_{x \to -3} \frac{(x^2 - 9)(1 - \sin^2(\pi x))}{(x - 3)^2 \cos(\pi x)}
\]

(1) to (2):  
\[
(2) = \lim_{x \to -3} \frac{x^2 - 9}{(x - 3)^2} \cdot \frac{1 - \sin^2(\pi x)}{\cos(\pi x)}
\]

(2) to (3):  
\[
(3) = \lim_{x \to -3} \frac{1}{1} \cdot \frac{1 - \sin^2(\pi x)}{\cos(\pi x)}
\]

(3) to (4):  
\[
(4) = \lim_{x \to -3} -\frac{\cos^2(\pi x)}{\cos(\pi x)}
\]

(4) to (5):  
\[
(5) = \lim_{x \to -3} -\frac{\cos(\pi x)}{1}
\]

(5) to (6):  
\[
(6) = -\frac{\pi \cos(x)}{1}
\]

(6) to (7):  
\[
(7) = -\pi \cos(-3)
\]

(b) If Scotty made one or more errors in his work, correctly evaluate \( \lim_{x \to -3} \frac{(x^2 - 9)(1 - \sin^2(\pi x))}{(x - 3)^2 \cos(\pi x)} \).

If there are no errors in the work above, draw a smiley face and write ‘Great job Scotty!’
5. Consider the story below:

While playing, Abby runs a circle around her house for 3 minutes. She then runs inside her
house to grab her coat at the center of the house, which takes half a minute, and then walks toward
her friend Liam’s house. After walking for 1 minute, Abby stops and decides that she should ride her
bike. She walks back home to get her bike, and then bikes to Liam’s house, which is 5 minutes away.

Draw a graph representing the above scenario of the story, where Abby’s distance from the center of
the house $d(t)$ is the vertical axis, and the horizontal axis is time $t$. Be sure to label the horizontal
axis, vertical axis, and events in the story below. To label each event in the story, use the corre-
sponding letter below. (For example, label (a) on the graph to the part that corresponds to Abby
running around her house.)

(a) Abby running around her house
(b) Abby running back into her house to grab her coat
(c) Abby walking to Liam’s house
(d) Abby walking back to get her bike
(e) Abby biking to Liam’s house