1. Evaluate \( \int \frac{\sin(\sqrt{x})}{\sqrt{x^2}} \, dx \). Be sure to show all work for full credit.

2. Consider \( \int \sin(x) \cos(x) \, dx \).
   
   (a) Evaluate the integral using substitution with \( u = \sin(x) \).
   
   (b) Evaluate the integral using substitution with \( u = \cos(x) \).
   
   (c) Use the Pythagorean Identity to show that your results from parts (a) and (b) are equivalent.

\[ \sin^2 x + \cos^2 x = 1 \]

So \( \sin^2 x = 1 - \cos^2 x \), thus we can rewrite (a) as \( \frac{1}{2} (1 - \cos^2 x) + C \) or \( -\frac{1}{2} \cos^2 x + \left(\frac{1}{2} + C\right) \) (constant).

On the other hand, \( \cos^2 x = 1 - \sin^2 x \), thus we can rewrite (b) as \( -\frac{1}{2} (1 - \sin^2 x) + K \) or \( \frac{1}{2} \sin^2 x + \left(-\frac{1}{2} + K\right) \) (constant).
3. Consider $\int_{-4}^{0} \sqrt{16-x^2} \, dx$ to answer the following

(a) Why do the substitution methods we have learned in class not ‘work’ for evaluating the integral?

if we were to let $u = 16 - x^2$
$\, du = -2x \, dx$
\[
\text{i.e. would need } \int x\sqrt{16-x^2} \, dx \text{ for } u\text{-sub to work.}
\]
and there is no additional ‘$x$’ in the integral.

(b) Find the exact value of the integral using geometry. Justify your answer. The use of technology is not accepted for this problem.

\[
\frac{1}{4} \pi (4)^2 = (4\pi)
\]

4. Consider the region bounded by $F(x) = x + 4$, $G(x) = (x - 2)^2$, and $x = 4$ as shown in the graph below.

(a) Write the integral that will give the area of the shaded region.

(b) Evaluate the integral you wrote in part (a).

\[
\int_{0}^{4} (x+4- (x^2-4x+4)) \, dx
\]
\[
= \int_{0}^{4} -x^2 +5x \, dx = \left[ -\frac{x^3}{3} + \frac{5}{2} x^2 \right]_{0}^{4}
\]
\[
= \left[ -\frac{(4)^3}{3} + \frac{5}{2} (4)^2 \right] - 0
\]
\[
= -\frac{64}{3} + 40 = \frac{56}{3}
\]
5. Find the combined area of the shaded regions illustrated in the graphic below:

\[ \int_{-2}^{0} \left( \frac{1}{9} x^3 + 2 \right) - (x+2) \, dx + \int_{0}^{3} (x+2) - \left( \frac{1}{4} x^3 + 2 \right) \, dx \]

\[ \int_{-2}^{0} \frac{1}{9} x^3 - x \, dx + \int_{0}^{3} x - \frac{1}{4} x^3 \, dx \]

\[ \left( \frac{1}{36} x^4 - \frac{1}{2} x^2 \right) \bigg|_{-2}^{0} + \left( \frac{1}{2} x^2 - \frac{1}{36} x^4 \right) \bigg|_{0}^{3} \]

\[ \left[ (0) - \left( \frac{16}{36} - 2 \right) \right] + \left[ \left( \frac{9}{2} - \frac{81}{36} \right) - (0) \right] \]

\[ \frac{137}{36} \]
6. All parts of this problem are based on the function \( f(x) = \frac{1}{x^P} \) for \( P > 0 \). The graph of \( f(x) \) is provided below.

Let \( P \neq 1 \). Set up and evaluate an integral for the area between \( f(x) \) and the \( x \)-axis for \( x = 1 \) to \( x = \circ \) for some constant \( \circ > 1 \).

\[
\int_{1}^{\circ} \frac{1}{x^P} \, dx = \int_{1}^{\circ} -x^{-P} \, dx = \left. \frac{x^{-P+1}}{-P+1} \right|_{1}^{\circ} = \frac{1}{-P+1} - \frac{1}{-P+1} \]

Simplify your result. Fill in the boxes with the appropriate exponent on \( \circ \) and the subtracted constant for your final answer:

\[
\frac{1}{-P+1} \left( \frac{-P+1}{-P+1} - 1 \right) \]

(a) What happens if \( P = 1 \)?

The result above is undefined.

(b) Let \( P > 1 \). Then the quantity \((-P + 1)\) is positive / negative (circle one).

What happens to the area as \( \circ \to \infty \)? (i.e. Take the limit as \( \circ \to \infty \))

Note: If \(-P + 1 = -\#\), then think about \( \circ^{-\#} = \frac{1}{\circ^{\#}} \) which \( \to 0 \) as \( \circ \to \infty \)

(c) Let \( 0 < P < 1 \). Then the quantity \((-P + 1)\) is positive / negative (circle one).

What happens to the area as \( \circ \to \infty \)?

Note: If \(-P + 1 = +\#\), then think about what happens for \( \circ^\# \)

So \( \circ^\# \to \infty \) as \( \circ \to \infty \)