2. Suppose that \( f(x) \) denotes a function that is continuous for all real numbers. The statement below is true sometimes. Give an example of a function for which it holds true and an example of a function for which it does not hold true. Explain your reasoning. Provide your answers by filling in the table below:

\[
\int_a^b f(x) \, dx \text{ will give the total area enclosed by the curve and the } x\text{-axis.}
\]

<table>
<thead>
<tr>
<th>Example of True</th>
<th>Example of False</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Diagram of a function above the x-axis" /></td>
<td><img src="image2" alt="Diagram of a function below the x-axis" /></td>
</tr>
</tbody>
</table>

Why is the statement true for your example?

If \( f(x) \) is on \( \frac{b}{2} \) or completely above the x-axis, then \( \int_a^b f(x) \, dx \) will give the total area enclosed by \( f(x) \) and the x-axis.

Why is the statement false for your example?

If \( f(x) \) is above \( \frac{b}{2} \) below the x-axis, then \( \int_a^b f(x) \, dx \) will not give the total area enclosed by \( f(x) \) and the x-axis.
(b) Estimate \( \int f(x) \, dx \) on \([-3, 2]\) using \( n = 5 \) subintervals of equal length and right endpoints. Be sure to draw the 5 rectangles you used on the graph provided and show all of the terms you included in your computation.

\[
\int_{-3}^{2} f(x) \, dx \approx (1)(8) + (1)(6) + 0 + (1)(-4) + 0 = 8 + 6 - 4 = 10
\]

There are 5 rectangles, so 5 terms should be written.
(c) Find an upper estimate for the value of \( \int_{-3}^{2} f(x) \, dx \) using \( n = 5 \) subintervals of equal length. Be sure to draw the 5 rectangles you used on the graph provided and show all of the terms you included in your computation. (Notice \( f(x) \) obtains a maximum at \( x = -1.786 \))

\[
\int_{-3}^{2} f(x) \, dx \approx 1(8) + 1(8.209) + 1(6) + 0 + 0 = 22.209
\]

(e) What is different about the way you draw the rectangles when you are finding an upper estimate or lower estimate compared to how you draw the rectangles when using right or left endpoints to approximate \( \int_{-3}^{2} f(x) \, dx \)? Describe in 2-5 sentences.

When computing the upper sum or lower sum we had to consider the entire subinterval to find the largest or smallest function value. This can end up being a combination of right and left endpoints or other values in the subinterval.
\( f(x) = x^3 + x^2 - 6x \)

(f) \( \int_{-3}^{2} f(x) \, dx = \frac{125}{12} \).

In terms of the graph of \( f \) above on the interval \([-3, 2]\), describe what the number \( \frac{125}{12} \) means.

It is the difference between the area of Region A & Region B. Because Region A is bigger & above the x-axis, the value is positive.

Could also say that \( \frac{125}{12} \) is the result of subtracting the area of Region B from the area of Region A.

(g) \( \int_{-3}^{0} f(x) \, dx - \int_{0}^{2} f(x) \, dx = \frac{63}{4} - \frac{16}{3} = \frac{253}{12} \).

In terms of the graph of \( f \) above on the interval \([-3, 2]\), describe what the number \( \frac{253}{12} \) means.

The total area enclosed by \( f(x) \) & the x-axis.

\( \int_{-3}^{0} f(x) \, dx \) gives the area for region A.

\( -\int_{0}^{2} f(x) \, dx \) translates region B into a positive value so we can interpret this as area.

\( \frac{253}{12} \) represents the total area between \( f(x) \) and the x-axis.
4. The Riemann definite integral \( \int_a^b f(x)dx \) is defined as a limit of Riemann sums. The definite integral can be *approximated* by a finite sum:

\[
\int_a^b f(x)dx \approx f(c_1)\Delta x_1 + f(c_2)\Delta x_2 + \ldots + f(c_{n-1})\Delta x_{n-1} + f(c_n)\Delta x_n
\]

where \( \Delta x_1 = (x_1 - x_0), \Delta x_2 = (x_2 - x_1), \ldots, \Delta x_{n-1} = (x_{n-1} - x_{n-2}), \Delta x_n = (x_n - x_{n-1}), \)

and \( c_i \in [x_{i-1}, x_i] \) for \( i=1,2, \ldots, n. \)

A function \( y = f(x) \) defined on the interval \( [a,b] = [-1,2] \) is shown in the figure.

(a) Explain in words how to interpret \( x_0, x_1, x_2, \ldots, x_{n-1}, x_n. \) (Label these on the x-axis in the figure above with \( n = 6. \))

Each \( x_i \) is an endpoint of a subinterval.

(b) Explain in words how to interpret \( f(c_1), f(c_2), \ldots, f(c_{n-1}), f(c_n). \)

\( f(c_i) \) is the height of the \( i^{th} \) rectangle.

(c) In the figure above, draw in the rectangles that correspond to the finite sum when \( n = 6 \) and then label the widths of the bases and heights of your rectangles using the notation provided in the problem statement. Answers will vary.