2. Use \( f(x) = \begin{cases} x^2 - \frac{4}{x} - 2 & x < 2 \\ ax^2 - bx + 1 & 2 \leq x < 3 \\ 4x - a + b & x \geq 3 \end{cases} \)

Find the value(s) of \( a \) and \( b \) that make \( f \) continuous everywhere. Explain how you know that the value(s) you found make \( f \) continuous everywhere.

\[
\begin{align*}
f(2) &= a(2)^2 - b(2) + 1 = 4a - 2b + 1 = \lim_{x \to 2} f(x) \\
\lim_{x \to 2} f(x) &= (2)^2 - \frac{4}{2} - 2 = 4 - 2 - 2 = 0
\end{align*}
\]

Need \( 4a - 2b + 1 = 0 \)

\[
\begin{align*}
f(3) &= 4(3) - a + b = 12 - a + b = \lim_{x \to 3} f(x) \\
\lim_{x \to 3} f(x) &= a(3)^2 - b(3) + 1 = 9a - 3b + 1
\end{align*}
\]

Need \( 12 - a + b = 9a - 3b + 1 \)

\[
\begin{align*}
4a - 2b &= -1 \\
-10a + 4b &= -11
\end{align*}
\]

\[
\begin{align*}
2a &= -13 \\
a &= \frac{-13}{2} = 6.5 \\
b &= \frac{27}{2} = 13.5
\end{align*}
\]

3. Evaluate the following limit (BE SURE TO SHOW ALL WORK):

\[
\lim_{t \to 1} \frac{t^3 - 2t^2 + t}{t - 1} = \lim_{t \to 1} \frac{t(t^2 - 2t + 1)}{t - 1} = \lim_{t \to 1} \frac{t(t-1)^2}{t - 1} = \lim_{t \to 1} \frac{t(t-1)}{t - 1} = \lim_{t \to 1} t(t-1) = 0
\]

4. Determine the vertical asymptote(s) of \( g(x) \).

Note that \( g(x) \) is undefined at \( x = 1 \). This is not enough evidence of a VA, since other things (hole, jump, etc.) could be happening at \( x = 1 \).

Need to test using the definition,

\[
\lim_{x \to 1^-} g(x) = \lim_{x \to 1^-} \frac{|x|}{x - 1} \not\to 2
\]

Thus \( \lim_{x \to 1^-} g(x) = -\infty \)

Note: We can stop here. This is sufficient evidence of a VA.

But let’s go ahead and look at \( \lim_{x \to 1^+} g(x) = \lim_{x \to 1^+} \frac{1 + |x|}{x - 1} \not\to 2 \)

Thus \( \lim_{x \to 1^+} g(x) = +\infty \)