1. For the function \( f(x) = x^4 + x^2 \) on \((-\infty, \infty)\),
   (a) \( x = 0 \) is both a critical point and inflection point.
   (b) \( x = 0 \) is a critical point.
   (c) \( x = 0 \) is an inflection point.
   (d) None of the above.
   Justify your work:

2. The function \( f(x) = \sin\left(\frac{\pi x}{2}\right) + \cos\left(\frac{\pi x}{2}\right) \) on \([-1, 5]\) is decreasing for all \( x \) in the interval (circle all correct answers)
   (a) \((-1, \frac{1}{2})\)    (c) \(\frac{1}{2}, \frac{5}{2}\)   (e) \(-\frac{1}{2}, \frac{3}{2}\)   (g) \(\frac{3}{2}, 5\)
   (b) \((-1, -\frac{1}{2})\)    (d) \(\frac{5}{2}, \frac{9}{2}\)   (f) \(\frac{3}{2}, \frac{7}{2}\)   (h) \(\frac{7}{2}, 5\)
   Justify your work:
3. The function \( f(x) = \sin\left(\frac{\pi x}{2}\right) + \cos\left(\frac{\pi x}{2}\right) \) on \([-1, 5]\) is concave up for all \( x \) in the interval (circle all correct answers)

- (a) \((-1, \frac{1}{2})\)
- (b) \((-1, -\frac{1}{2})\)
- (c) \(\left(\frac{1}{2}, \frac{5}{2}\right)\)
- (d) \(\left(\frac{5}{2}, \frac{9}{2}\right)\)
- (e) \(-\frac{1}{2}, \frac{3}{2}\)\)
- (f) \(\left(\frac{3}{2}, \frac{7}{2}\right)\)
- (g) \(\left(\frac{5}{2}, 5\right)\)
- (h) \(\left(\frac{7}{2}, 5\right)\)

Justify your work:

4. Suppose that \( H'(x) \) is continuous, increasing, and negative on \([0, 1]\). Then \( H(x) \) is

- (a) increasing and concave down on \((0, 1)\).
- (b) decreasing and concave down on \((0, 1)\).
- (c) increasing and concave up on \((0, 1)\).
- (d) decreasing and concave up on \((0, 1)\).
- (e) None of the above.

Justify your answer:
5. Find two positive numbers such that their product is 36 and the sum of 4 times the first number plus the second number is a minimum. Use calculus to solve and justify your work.

6. A smokestack deposits soot on the ground with a concentration inversely proportional to the square of the distance from the stack. With two smokestacks 20 miles apart, the concentration, $C$, of the combined deposits on the line joining them, at a distance $x$ from one stack, is given by

$$C(x) = \frac{k_1}{x^2} + \frac{k_2}{(20 - x)^2}$$

Where $k_1$ and $k_2$ are positive constants which depend on the quantity of smoke each stack is emitting, with $k_1 = 7k_2$.

Write $C(x)$ in terms of $k_2$ (i.e. fill in the empty box): 

$$C(x) = \frac{\phantom{k_2}}{x^2} + \frac{k_2}{(20 - x)^2}$$

Find the point on the line joining the stacks where the concentration, $C$, of the deposits is a minimum.

I used the first / second (circle one) derivative test to show that $x = \phantom{\\text{miles from the smokestack}}$ miles from the smokestack will result in a minimum value of concentration $C = \phantom{\text{minimum value}}$. 


7. A crate of pumpkins with a fixed weight \( W \) is dragged along a horizontal plane at a constant velocity by a force acting along a rope attached to the crate. If the rope makes an angle \( \theta \) with the plane, then the magnitude of the force is

\[
F(\theta) = \frac{\mu W}{\mu \sin(\theta) + \cos(\theta)}
\]

where \( \mu \) is a positive constant called the coefficient of friction.

(a) Draw a diagram of the situation.

(b) What values of \( \theta \) make sense in this circumstance?

\[
\quad \leq \theta < \quad
\]

(c) Find \( F'(\theta) \):

(d) Show \( \theta = \tan^{-1}(\mu) \) is a critical point of \( F(\theta) \).
Problem 7 is continued on this page.

(e) 

\[ F''(\theta) = \mu W \left( \frac{2(\mu \cos(\theta) - \sin(\theta))^2}{(\mu \sin(\theta) + \cos(\theta))^3} + \frac{1}{\mu \sin(\theta) + \cos(\theta)} \right) \]

Recall that the domain is \( \_ \_ \_ \leq \theta < \_ \_ \_ \)

\[ \frac{1}{\mu \sin(\theta) + \cos(\theta)} \] is negative / zero / positive (circle one). Justify your answer in 1-2 sentences.

Recall that \( \mu \cos(\theta) - \sin(\theta) = 0 \) at the critical point \( \theta = \tan^{-1}(\mu) \).

Use the second derivative test with the above information to determine if there is a local maximum or local minimum at the critical point \( \theta = \tan^{-1}(\mu) \).

There is a local maximum / minimum (circle one) at \( \theta = \tan^{-1}(\mu) \).

Note that on this problem testing with the first derivative would have been quite difficult given that \( \mu \) is an unknown parameter. The second derivative, however, was much easier to work with and analyze.