Written HW 08 - Due Friday, October 20

Name: ____________________________________________

1. (a) Let \( h(x) = \frac{x^2 - 4}{x^2 + 9} \)

What is the domain of \( h(x) \)?

(b) Use the first derivative to determine the critical point(s) of \( h(x) \).

(c) Use the first derivative to determine when the graph of \( h(x) \) is increasing and when the graph of \( h(x) \) is decreasing.

\( h(x) \) is increasing on the interval: _______________________

\( h(x) \) is decreasing on the interval: _______________________

2. Below are the graphs of a function and its derivatives: \( f(x), f'(x), f''(x) \). Identify which function is which. Give reasons for your answers in sentences.

Graph 1 is
\[
\begin{align*}
\frac{f(x)}{f'(x)} / f''(x)
\end{align*}
\]
(circle one)

Graph 2 is
\[
\begin{align*}
\frac{f(x)}{f'(x)} / f''(x)
\end{align*}
\]
(circle one)

Graph 3 is
\[
\begin{align*}
\frac{f(x)}{f'(x)} / f''(x)
\end{align*}
\]
(circle one)

Justify your answers
Give reasons for your answers in sentences. Your explanations should include a discussion of slope and concavity with regard to each graph.
3. (a) Let $f(x) = \frac{a}{x} + x$, where $a$ is a positive constant.

What is the domain of $f(x)$?

(b) Use the first derivative to determine the critical point(s) of $f(x)$.

(c) Use the first derivative to determine when the graph of $f(x)$ is increasing and when the graph of $f(x)$ is decreasing.

$f(x)$ is increasing on the intervals: ________________________________

$f(x)$ is decreasing on the intervals: ________________________________
4. (a) Let \( g(x) = \frac{1}{\sqrt{x}} + 4bx \), where \( b \) is a positive constant.

What is the domain of \( g(x) \)?

(b) Use the first derivative to determine the critical point(s) of \( g(x) \).

(c) Use the second derivative test to determine if a local maximum or local minimum occurs at each critical point.
5. Using calculus, find the values of $c$ and $d$ so that the function $f(x) = -8x^2 - 2cx + d$ has a local maximum at the point $(\frac{1}{2}, 4)$.

6. In the axes provided, sketch the graph of a function, $f$, that has the following properties.

- $x = -2$ is a vertical asymptote
- $\lim_{x \to -\infty} f(x) = 3$
- $f'(0) = 0$ and $f'(5) = 0$
- $f''(5) = 0$
- $f''(2)$ is undefined
- $f'(x) > 0$ on $(0, 2)$
- $f'(x) < 0$ on $(-\infty, -2) \cup (-2, 0) \cup (2, 5) \cup (5, \infty)$
- $f''(x) > 0$ on $(-2, 2) \cup (2, 5)$
- $f''(x) < 0$ on $(-\infty, -2) \cup (5, \infty)$