1. Let \( g(x) = \sqrt{2} \cdot f(x) \) and \( h(x) = f(\sin(x)) \).

The table below provides values of \( f(x) \) and \( f'(x) \) at given \( x \)-values.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>( \pi )</th>
<th>( \frac{\sqrt{2}}{2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>8</td>
<td>( \frac{\pi}{3} )</td>
<td>(-\pi)</td>
</tr>
<tr>
<td>( f'(x) )</td>
<td>(2\pi)</td>
<td>-1</td>
<td>2</td>
</tr>
</tbody>
</table>

(a) Find \( g'(x) \).

(b) Find \( g'(0) \).

(c) Find the equation of the tangent line to \( g(x) \) at \( x = 0 \).

(d) Find \( \frac{dh}{dx} \bigg|_{x=\pi/4} \).
2. Let \( r(x) = (H \circ G)(x) \) and \( t(x) = (G \circ H)(x) \).
Use the graphs of the functions \( G(x) \) and \( H(x) \) below to find the following derivatives. You may need to estimate the slope of graphs at particular points. If an answer is undefined, state why in 1-2 sentences.

(a) \( r'(0) \)

(b) \( t'(1) \)
3. Calculate the indicated derivatives by using the Differentiation Rules (Theorems). Answers must be accompanied by supporting work that shows how you calculated the derivative. **You do not need to simplify your answers on these problems.** If you do simplify an answer, you must simplify correctly. Answers will be scored right or wrong. Don’t expect partial credit for incorrect answers or answers with no supporting work.

(a) \( g(x) = \frac{\cos(\pi x)}{\sqrt{x}} \)

(b) \( h(r) = \sqrt[3]{7r^5} - r + \frac{\pi^2}{r} \)

(c) \( f(x) = \frac{1}{6} \tan(2\pi x) \sin\left(\frac{2}{x}\right) \)
4. Suppose that \( f(x) \) denotes a function defined for all real numbers.

Indicate whether each of the following statements is True or False. If the statement is true, explain how you know it is true. If it is false, give a counterexample and explain why it is a counterexample. (A counterexample is an example of a function for which the “if” part of the statement is true, but the “then” part is false.) A graph with an explanation in words can be used as a counterexample.

(a) If \( f(x) \) is continuous at \( x = 3 \), then \( f'(3) \) must exist.

(b) If \( f(x) \) is continuous at \( x = 4 \), then \( f(4) \) must exist.

(c) If \( f'(7) \) does not exist, \( f(x) \) must not be continuous at \( x = 7 \).
5. Use \( f(x) = |x + 1| \) and \( g(x) = (x + 1)^{2/3} \) to answer the following:

(a) \( f(x) \) is / is not (circle one) continuous at \( x = -1 \).

(b) \( g(x) \) is / is not (circle one) continuous at \( x = -1 \).

(c) Although neither function \( f(x) \) nor \( g(x) \) is differentiable at \( x = -1 \), the slopes of the curves are not behaving in the same way. What is happening with the slopes of each function on both sides of \( x = -1 \)?

**Description of \( f(x) \):**

\[
\lim_{h \to 0^-} \frac{f(-1 + h) - f(-1)}{h} = \text{__________}. \text{ So the slopes on the left side of } x = -1 \text{ are:} \\
\text{(provide a brief description about the slope of } f(x) \text{ on the left side of } x = -1)
\]

\[
\lim_{h \to 0^+} \frac{f(-1 + h) - f(-1)}{h} = \text{__________}. \text{ So the slopes on the left side of } x = -1 \text{ are:} \\
\text{(provide a brief description about the slope of } f(x) \text{ on the right side of } x = -1)
\]

**Description of \( g(x) \):**

\[
\lim_{h \to 0^-} \frac{g(-1 + h) - g(-1)}{h} = \text{__________}. \text{ So the slopes on the left side of } x = -1 \text{ are:} \\
\text{(provide a brief description about the slope of } g(x) \text{ on the left side of } x = -1)
\]

\[
\lim_{h \to 0^+} \frac{g(-1 + h) - g(-1)}{h} = \text{__________}. \text{ So the slopes on the left side of } x = -1 \text{ are:} \\
\text{(provide a brief description about the slope of } g(x) \text{ on the right side of } x = -1)
\]

(d) Use the results of your discussion in (a) to come up with a simple and concise way to clearly distinguish between a cusp and a corner. i.e. How can one determine, in general, if a function has a cusp or corner?