Written HW 03 - Due Tuesday, September 12

Name: ________________________________

1. Use \( g(x) = \begin{cases} \sqrt{2} \sin \left( \frac{x}{2} \right) & \text{if } x < 4 \\ J & \text{if } x = 4 \\ (x - 5)^2 - 1 & \text{if } x > 4 \end{cases} \) to answer the following.

Be sure to show ALL WORK AND STEPS. When evaluating parts (a), (b), and (c) below, be sure to be clear about which piece of the function you are using. For example, if you are using \((x - 5)^2 - 1\), then state that you are using piece 3.

(a) Determine the value of \( g(4) \):

(b) Determine the value of \( \lim_{x \to 4^-} g(x) \):

(c) Determine the value of \( \lim_{x \to 4^+} g(x) \):

(d) In order for \( g(x) \) to be continuous at \( x = 4 \), what must the value of \( J \) be?

Explain your reasoning in 1-2 complete sentences and use the mathematical definition of continuity.
2. Use \( f(x) = \begin{cases} \frac{3}{x} - 2 & x < -1 \\ ax^2 - x + c & -1 \leq x < 4 \\ ax + 3 & x \geq 4 \end{cases} \) to answer the following:

Find the value(s) of \( a \) and \( c \) that make \( f \) continuous everywhere. Your work should include limit calculations. If your work does not include limit calculations, points will be taken off on this problem. Explain how you know that the value(s) you found make \( f \) continuous everywhere.

3. Evaluate the following limits (BE SURE TO SHOW ALL WORK). Be sure to show all work OR if the limit doesn’t exist/is infinite, explain how you know.

\[
\lim_{t \to -3} \frac{t + 3}{t^3 + 6t^2 + 9t}
\]

\[
\lim_{t \to -3} \frac{t^3 + 6t^2 + 9t}{t + 3}
\]
4. Use \( g(x) = \frac{2 + |x|}{x + 2} \) to answer the following:

(a) Determine the horizontal asymptote(s) of \( g(x) \). (Your answer should be in the form of \( y = \)).
   Explain your answer clearly using the limit definition of horizontal asymptote. If there are no horizontal asymptotes, draw a smiley face in the space below.

(b) Determine the vertical asymptote(s) of \( g(x) \). (Your answer should be in the form of \( x = \)).
   Explain your answer clearly using the limit definition of vertical asymptote. If there are no vertical asymptotes, draw a smiley face in the space below.

(c) Draw the graph of \( g(x) \) on the interval \([-1, 3]\).

Fill-In-The-Blanks: On the interval \([-1, 3]\)

\( g \) has an absolute maximum of

\( y = \underline{\phantom{0}} \) for \( x \underline{\phantom{0}} \)

and

\( g \) has an absolute minimum of

\( y = \underline{\phantom{0}} \) for \( x \underline{\phantom{0}} \).

(d) What can be said about the absolute extrema of \( g \) if we consider it on its entire domain rather than \([-1, 3]\)? Provide a justification for your answer.
5. Indicate whether each of the following statements is True or False. If the statement is true, explain how you know it’s true. If it is false, give a counterexample and explain why it is a counterexample. (A counterexample is an example that shows the statement is false.)
(a) Every rational function has at least one vertical asymptote.

(b) If \( f(-1) = 2 \) and \( f(1) = -2 \), then there is an \( x \)-value \( c \) between \(-1\) and \(1\) where \( f(c) = 0 \).

(c) If \( f(x) \) is defined on \([-3, 4]\), then \( f \) must attain an absolute maximum on \([-3, 4]\).