1. Use \( g(x) = \begin{cases} \sqrt{2} \sin \left( \frac{\pi x}{2} \right) & \text{if } x < 4 \\ J & \text{if } x = 4 \\ (x - 5)^2 - 1 & \text{if } x > 4 \end{cases} \) to answer the following.

Be sure to show ALL WORK AND STEPS. When evaluating parts (a), (b), and (c) below, be sure to be clear about which piece of the function you are using. For example, if you are using \( (x - 5)^2 - 1 \), then state that you are using piece 3.

(a) Determine the value of \( g(4) \):

\[ g(4) = J \]

Using the 2nd piece

(b) Determine the value of \( \lim_{x \to 4^-} g(x) \):

\[ \lim_{x \to 4^-} \sqrt{2} \sin \left( \frac{\pi x}{2} \right) = \sqrt{2} \cdot \sin (2\pi) = 0 \]

Using the 1st piece

(c) Determine the value of \( \lim_{x \to 4^+} g(x) \):

\[ \lim_{x \to 4^+} (x - 5)^2 - 1 = (4 - 5)^2 - 1 = 1 - 1 = 0 \]

Using the 3rd piece

(d) In order for \( g(x) \) to be continuous at \( x = 4 \), what must the value of \( J \) be?

Explain your reasoning in 1-2 complete sentences and use the mathematical definition of continuity.

Need \( \lim_{x \to 4^-} g(x) = g(4) \). Since \( \lim_{x \to 4^-} g(x) = 0 = \lim_{x \to 4^+} g(x) \), we can say \( \lim_{x \to 4} g(x) = 0 \). Thus to be continuous, need \( g(4) = 0 \), so \( J = 0 \).
4. Use $g(x) = \frac{2 + |x|}{x + 2}$ to answer the following:

(a) Determine the horizontal asymptote(s) of $g(x)$. (Your answer should be in the form of $y =$).

Explain your answer clearly using the limit definition of horizontal asymptote. If there are no horizontal asymptotes, draw a smiley face in the space below.

$$\lim_{x \to \infty} \frac{2 + |x|}{x + 2} \text{ since } x \to \infty, \quad |x| = x \quad \text{giving} \quad \lim_{x \to \infty} \frac{2 + x}{x + 2} = 1$$

$$\lim_{x \to -\infty} \frac{2 + |x|}{x + 2} \text{ since } x \to -\infty, \quad |x| = -x \quad \text{giving} \quad \lim_{x \to -\infty} \frac{2 - x}{(x + 2) \frac{1}{x}} = \lim_{x \to -\infty} \frac{2x - 1}{1 + \frac{x}{x}} = -1$$

(b) Determine the vertical asymptote(s) of $g(x)$. (Your answer should be in the form of $x =$).

Explain your answer clearly using the limit definition of vertical asymptote. If there are no vertical asymptotes, draw a smiley face in the space below.

$$\lim_{x \to -2} \frac{2 + |x|}{x + 2} \quad \text{Note: when looking at } x \to -2, \quad |x| = -x$$

$$\lim_{x \to -2} \frac{2 - x}{x + 2} \quad \text{Left-hand} \quad \lim_{x \to -2^+} \frac{2 - x}{x + 2} \quad \text{as } x \to -2^+ \to 0 \text{ and is negative}$$

$$\lim_{x \to -2} \frac{2 - x}{x + 2} \quad \to -\infty \quad \text{positive as } x \to -2^+ \text{ and } \quad 2 - x \to 4 \quad \text{as } x \to -2^- \text{.}$$

(c) Draw the graph of $g(x)$ on the interval $[-1, 3]$.

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Fill-In-The-Blanks: On the interval $[-1, 3]$

$g$ has an absolute maximum of

$y = \frac{3}{2}$ for $x = -1$

and

$g$ has an absolute minimum of

$y = \frac{1}{2}$ for $x \in [0, 3]$.

(d) What can be said about the absolute extrema of $g$ if we consider it on its entire domain rather than $[-1, 3]$? Provide a justification for your answer.

$g(x)$ is continuous on its domain which is $(-\infty, -2) \cup (-2, \infty)$.

However, this is not a closed interval, so we cannot apply the EVT.
(b) If \( f(-1) = 2 \) and \( f(1) = -2 \), then there is an \( x \)-value \( c \) between \(-1\) and \(1\) where \( f(c) = 0 \).

(c) If \( f(x) \) is defined on \([-3, 4]\), then \( f \) must attain an absolute maximum on \([-3, 4]\).