2. Each of the following statements are sometimes true and sometimes false. Draw a graph of a function for both cases. Provide a 1-2 sentence explanation of why your graph satisfies the true statement. Provide a 1-2 sentence explanation of why your graph satisfies the false statement.

(a) If \( \lim_{x \to 1} f(x) \) exists, then \( \lim_{x \to 1} f(x) = f(1) \)

Graph for when statement is true: \[
\begin{align*}
\text{Graph for when statement is false: }
\end{align*}
\]

Explanation: \( f(1) \) may or may not exist. The value of the limit does not influence the function value (or its existence).

(b) If \( f(-3) \) exists, then \( \lim_{x \to -3} f(x) \) must exist.

Graph for when statement is true: \[
\begin{align*}
\text{Graph for when statement is false: }
\end{align*}
\]

Explanation: \( f(-3) \) exists and the limit exists.
4. Scotty needs to evaluate the limit of an oscillating function. His work is below.

(a) Evaluate Scotty’s work. If errors occur from one step to the next, state the errors on the line provided. If there is no error in the step, draw a smiley face.

\[
\lim_{x \to -3} \frac{(x^2 - 9)(1 - \sin^2(\pi x))}{(x - 3)^2 \cos(\pi x)}
\]

(1) \[
\lim_{x \to -3} \frac{(x^2 - 9)}{(x - 3)^2} \cdot \frac{(1 - \sin^2(\pi x))}{\cos(\pi x)}
\]

(2) \[
\lim_{x \to -3} \frac{(x^2 - 9)}{(x - 3)^2} \cdot \frac{(1 - \sin^2(\pi x))}{\cos(\pi x)}
\]

(3) \[
\lim_{x \to -3} 1 \cdot \frac{(1 - \sin^2(\pi x))}{\cos(\pi x)}
\]

(4) \[
\lim_{x \to -3} -\frac{\cos^2(\pi x)}{\cos(\pi x)}
\]

(5) \[
\lim_{x \to -3} -\frac{\cos(\pi x)}{1}
\]

(6) \[
-\frac{\pi \cos(x)}{1}
\]

(7) \[
-\pi \cos(-3)
\]

(1) to (2): \[
\begin{align*}
\text{to (2)}: & \quad \text{\(x^2 - 9 = (x-3)(x+3)\)} \\
\text{(2) to (3)}: & \quad \text{\(x^2 - 9 \neq (x-3)^2\)} \\
\text{(3) to (4)}: & \quad \text{\(\sin^2(\pi x) + \cos^2(\pi x) = 1\), so} \\
\text{(4) to (5)}: & \quad \text{\(\cos^2(\pi x) = 1 - \sin^2(\pi x)\)}
\end{align*}
\]

(5) to (6): \[
\text{\(-\cos(\pi x) \neq -\pi \cos x\). Cannot stop writing limit.}
\]

(6) to (7): \[
\text{Not true since limit wasn’t written in previous step.}
\]

(b) If Scotty made one or more errors in his work, correctly evaluate \[
\lim_{x \to -3} \frac{(x^2 - 9)(1 - \sin^2(\pi x))}{(x - 3)^2 \cos(\pi x)}
\]

If there are no errors in the work above, draw a smiley face and write ‘Great job Scotty!’

\[
\lim_{x \to -3} \frac{(x^2 - 9)(1 - \sin^2(\pi x))}{(x - 3)^2 \cos(\pi x)} = \frac{0 \cdot (1 - \sin^2(-3\pi))}{(-6)^2 \cos(-3\pi)}
\]

\[
= \frac{0}{36(-1)} = 0
\]
5. Consider the story below:

While playing, Abby runs a circle around her house for 3 minutes. She then runs inside her house to grab her coat at the center of the house, which takes half a minute, and then walks toward her friend Liam’s house. After walking for 1 minute, Abby stops and decides that she should ride her bike. She walks back home to get her bike, and then bikes to Liam’s house, which is 5 minutes away.

Draw a graph representing the above scenario of the story, where Abby’s distance from the center of the house $d(t)$ is the vertical axis, and the horizontal axis is time $t$. Be sure to label the horizontal axis, vertical axis, and events in the story below. To label each event in the story, use the corresponding letter below. (For example, label (a) on the graph to the part that corresponds to Abby running around her house.)

(a) Abby running around her house
(b) Abby running back into her house to grab her coat
(c) Abby walking to Liam’s house
(d) Abby walking back to get her bike
(e) Abby biking to Liam’s house