1. Use the graph of $f$ below to answer the following questions. Define $G(x) = \int_0^x f(t) \, dt$ for $0 \leq x \leq 6$.

(b) List all the critical points of $G(x)$.

where $G' = 0$/DNE + endpoints

$G' = f$

$x = 0, 1, 2, 3, 4, 5, 6$
2. Suppose \( f(t) \) is continuous and \( f(t) > 1 \) for all \( t \geq 0 \). Define functions \( F(x) \) and \( G(x) \) by the equations 
\[
F(x) = \int_0^x f(t)\,dt \quad \text{and} \quad G(x) = \int_5^x f(t)\,dt.
\]

(a) Which is larger, \( F(1) \) or \( F(2) \)? Explain how you know.

\[
\begin{align*}
\int_0^1 f(t)\,dt > 0 \quad \text{so} \quad F(2) > F(1),
\end{align*}
\]

Thus \( F(2) > F(1) \).

Or \( \int_0^1 f(t)\,dt = \int_0^2 f(t)\,dt - \int_2^1 f(t)\,dt \)

less area \quad more area


(b) Which is larger, \( F(6) \) or \( G(6) \)? Explain how you know.

\[
\begin{align*}
F(6) &= \int_0^6 f(t)\,dt \\
G(6) &= \int_5^6 f(t)\,dt \\
&= \int_0^6 f(t)\,dt - \int_0^5 f(t)\,dt \\
&= F(6) - G(6)
\end{align*}
\]

Thus \( F(6) > G(6) \).

(c) Explain why \( F(x) > F(0) \) for all \( x > 0 \).

\[
F(x) = \int_0^x f(t)\,dt > 0 \\
\quad \text{so} \quad F(x) \text{ is increasing} \quad \text{so} \quad F(x) > F(0) \quad \text{for} \quad x > 0
\]

(d) Explain why \( F(x) = G(x) + C \) for some constant \( C \).

\[
\begin{align*}
\int_0^x f(t)\,dt &= \int_0^5 f(t)\,dt + \int_5^x f(t)\,dt \\
&= G(x) + C
\end{align*}
\]

(e) Is \( C \) positive or negative? Explain how you know.

\[
C = \int_5^6 f(t)\,dt > 0 \quad \text{because} \quad f(t) > 0 \quad \text{and} \quad 5 > 0.
\]