Case 1: $r < 1$. Then $r^n \to 0$ as $n \to \infty$ because $|r| < 1$. Hence $\lim_{n \to \infty} r^n = 0$.

Case 2: $r = 1$. Since $r = 1$, then $r^n = 1$ for all $n$. Therefore $\lim_{n \to \infty} r^n = 1$.

Let $\epsilon > 0$. Let $N \in \mathbb{N}$ such that $|r^n - 1| < \epsilon$ whenever $n > N$. Then $\left| \frac{1}{n} \right| < \frac{\epsilon}{r^n - 1}$.

Why? Because $|r^n - 1| < \epsilon$ implies $n > \frac{\ln(\epsilon)}{\ln|r|}$ when $|r| > 1$.

Thus, there exists $N = \frac{\ln(\epsilon)}{\ln|r|}$ such that $\left| \frac{1}{n} \right| < \frac{\epsilon}{r^n - 1}$ whenever $n > N$.

Case 3: $r > 1$. Suppose $\lim_{n \to \infty} r^n = L$. Then $\epsilon > 0$ there exists $N \in \mathbb{N}$ such that $|r^n - L| < \epsilon$ whenever $n > N$.

Take $\ln$ of both sides of the equation on the right.

Then $\ln r^n < \ln L + \epsilon$.

or $\ln r < \frac{\ln L + \epsilon}{n}$.

This contradicts the fact that $|r^n - L| < \epsilon$ must hold for all $n > N$ (Assumption), as $\lim_{\infty} r^n$ does.

Case 4: $r = -1$. Then $r^n = (-1)^n$.

The sequence $r^n$ oscillates between $1$ and $-1$. Therefore, $\lim_{n \to \infty} r^n$ does not exist.

The subsequence $r^n$ converges to $-1$ and $r^n$ converges to $1$.

Let $a$ be any real number. All subsequences of $r^n$ converge to the same value.

Case 5: $r < -1$. Then $r^n < 0$. So $r^n$ is not convergent (as $t \to \infty$), since in Case 3.