(a) Consider the set (0, 1).
\[ S = \text{set of positive real numbers}, \quad \text{claim } \inf S \geq 0 \]
\[ \text{Pf: } \lim_{x \to 0^+} x < S \Rightarrow 0 \text{ is a lower bound} \]
\[ \text{of } S \Rightarrow \text{g.l.u.} S \geq 0. \quad [\text{If } c \text{ is a lower bound of any set } S, \text{ g.l.u.} S \geq c.] \]

(c) \[ S \subseteq \mathbb{R}, \quad S \text{ bold above and } B = \emptyset, \quad B \subseteq S. \]
\[ \text{Then } \sup B \leq \sup S. \text{ True.} \]
\[ \text{Pf: Let } \sup S = S^* \text{ and } \sup B = b^*. \]
\[ x \in B \Rightarrow x < S \Rightarrow x < S^* \quad (\text{BCS and } X^* \text{ is an upper bound}) \]
\[ \therefore x^* \text{ is an upper bound of } B. \]
\[ \therefore \text{l.u.b.} B \leq x^* \quad (\text{If } c \text{ is an upper bound of any set,}\]
\[ \text{the least upper bound of the set must be } \leq c.) \]

HW 6

#11, page 11, Fitz.

(a) Prove that the sum of a rational and an irrational is irrational.

(b) Prove that the product of two nonzero real numbers one rational and one irrational, is irrational.

Pf(a) Let \( r \) be a rational and \( i \) be an irrational. Assume false i.e., that \( r + i \) is rational. Then \( i = \frac{m}{n} - r \) is rational. This is a contradiction. 

so \( r + i \) is irrational. (If \( r \) is rational, \( r = \frac{P}{Q} \) so \( m - \frac{r}{n} = \frac{mQ - Pr}{nQ} \) \( m - \frac{r}{n} \) is the ratio of two integers i.e., rational)

(b) Let \( r \) be a rational and \( i \) be irrational. Assume false, that \( r \times i \) is rational. Then \( \frac{r}{r} = i \) is rational. This is a contradiction so \( r \times i \) is irrational.