HW 32 #2, p. 65 Fitzpatrick.

Prove \( x^9 + x^2 + 4 = 0 \), \( x \in \mathbb{R} \) has a soln.

Let \( f(x) = x^9 + x^2 + 4. \) \( f(-7) = \text{something neg} < 0. \)
\( f(7) > 0. \) Apply IV Thm with \( c = 0. \) \( \Rightarrow \exists x_0 \in (-7, 7) \)
\( s.t. \) \( f(x_0) = 0. \)

HW 33 #5, p. 65 Fitzpatrick.

\( h : [a, b] \rightarrow \mathbb{R}, \ g : [a, b] \rightarrow \mathbb{R} \) cont.. If \( h(a) \leq g(a) \) and
\( h(b) \geq g(b), \) then there exists \( x_0 \) s.t. \( h(x_0) = g(x_0), x_0 \in [a, b]. \)

Let \( f(x) = h(x) - g(x). \) \( h(a) \leq g(a) \Rightarrow f(a) \leq 0. \) \( h(b) \geq g(b) \)
\( \Rightarrow f(b) \geq 0. \)

If
\( f(a) = 0 \) or \( f(b) = 0, \) we are done — with \( x_0 = a \) or \( x_0 = b. \)

If neither \( f(a) = 0 \) nor \( f(b) = 0, \) then \( f(a) < 0, f(b) > 0 \)
\( f(a) < 0 < f(b) \) and we can apply the IV Thm with \( c = 0 \)
to get \( x_0 \in (a, b) \) s.t. \( f(x_0) = 0. \)
\( \Rightarrow h(x_0) = g(x_0). \)