(a) If the function $f + g : \mathbb{R} \to \mathbb{R}$ is cont., then the functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are cont.

False. Let $f(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$

and $g(x) = \begin{cases} -1 & x \geq 0 \\ 0 & x < 0 \end{cases}$.

Then $(f + g)(x) = 0$ for all $x$.

Clearly, $f + g$ is cont. at $x = 0$ but neither $f$ nor $g$ is cont. at $x = \epsilon$.

(b) If the function $f^2 : \mathbb{R} \to \mathbb{R}$ is cont., then so is $f : \mathbb{R} \to \mathbb{R}$.

False. Let $f(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$. Then $f^2(x) = 1$ for all $x$.

$f^2$ is clearly cont. at $x = 0$ and $f$ is not cont. at $x = 0$.

(c) If the functions $f + g : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are cont., then so is $f : \mathbb{R} \to \mathbb{R}$.

True. $f = (f + g) + (-g)$. Then by Prop 4.14 (probably almost all parts) $f$ is cont.

(d) Every fn. $f : \mathbb{N} \to \mathbb{R}$ is continuous.

True. For any $x \in \mathbb{N}$ choose $\delta = 1/2$. 
