HW 18. #2, p. 37, Fitzpatrick

Show that \( S = (0, \infty) \) is closed.

Suppose \( x_n \in S \) and \( x_n \to x \). Then \( x_n > 0 \) for each \( n \) and \( -x_n \to -x \). Then by the law of excluded middle as a part of proving that \( [0, \infty] \) is closed, we know that \( -x \geq 0 \). \( -x < 0 \) so \( x \in S = (0, \infty) \). Thus \( S = (0, \infty) \) is closed.

HW 19. #3, p. 37, Fitzpatrick

Show that \( x \in \mathbb{R} \Rightarrow \exists x_n^3 \) s.t. \( x_n \to x \). Denote the collection of rationals by \( \mathbb{Q} \).

Pf: The rationals are dense in \( \mathbb{R} \) by Prop 2.13. By Prop 3.13, for \( x \in \mathbb{R} \) there exists \( \exists x_n^3 \in \mathbb{Q} \) s.t. \( x_n \to x \), which is what we wanted to prove.

#4. By #3 above for \( x \in \mathbb{R} \) \( \exists x_n^3 \in \mathbb{Q} \) s.t. \( x_n \to x \).

Choose \( x = 1 \). Then \( x_n^3 \in \mathbb{Q} \), \( x_n \to 1 \notin \mathbb{Q} \). \( \mathbb{Q} \) is not closed.