

# M161, Final, Spring 2008

Name: \_\_\_\_\_

Section: \_\_\_\_\_

Instructor: \_\_\_\_\_

Time: 110 minutes. You may not use calculators on this exam

$$\begin{aligned} \frac{d}{dx} \sin(x) &= \cos(x), & \frac{d}{dx} \cos(x) &= -\sin(x), \\ \frac{d}{dx} \tan(x) &= \sec^2(x), & \frac{d}{dx} \csc(x) &= -\csc(x) \cot(x), \\ \frac{d}{dx} \sec(x) &= \sec(x) \tan(x), & \frac{d}{dx} \cot(x) &= -\csc^2(x), \end{aligned}$$

| Problem  | Points | Score |
|----------|--------|-------|
| 1        | 20     |       |
| 2ab      | 30     |       |
| 2cd      | 30     |       |
| 3        | 30     |       |
| 4        | 30     |       |
| 5        | 30     |       |
| 6        | 30     |       |
| $\Sigma$ | 200    |       |

$$\begin{aligned} \frac{d}{dx} \sinh(x) &= \cosh(x), & \frac{d}{dx} \cosh(x) &= \sinh(x), & \frac{d}{dx} \tanh(x) &= \operatorname{sech}^2(x), \\ \frac{d}{dx} \operatorname{asin}(x) &= \frac{1}{\sqrt{1-x^2}}, & \frac{d}{dx} \operatorname{acos}(x) &= -\frac{1}{\sqrt{1-x^2}}, & \frac{d}{dx} \operatorname{atan}(x) &= \frac{1}{1+x^2}, \\ \frac{d}{dx} \operatorname{acsc}(x) &= -\frac{1}{x\sqrt{x^2-1}}, & \frac{d}{dx} \operatorname{asec}(x) &= \frac{1}{x\sqrt{x^2-1}}, & \frac{d}{dx} \operatorname{acot}(x) &= -\frac{1}{1+x^2}, \\ \frac{d}{dx} \operatorname{asinh}(x) &= \frac{1}{\sqrt{1+x^2}}, & \frac{d}{dx} \operatorname{acosh}(x) &= \frac{1}{\sqrt{x^2-1}}, & \frac{d}{dx} \operatorname{atanh}(x) &= \frac{1}{1-x^2}, \\ \sin(2x) &= 2 \sin(x) \cos(x) & \int \ln x dx &= x \ln x - x + C & \int \sec(x) dx &= \ln(\sec(x) + \tan(x)) + C \\ \tan^2(x) + 1 &= \sec^2(x) & \cos^2(x) &= \frac{1 + \cos(2x)}{2} & \sin^2(x) &= \frac{1 - \cos(2x)}{2} \end{aligned}$$

Taylor series of the function  $f(x)$  about  $x = a$ :

$$f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

**Theorem** (The Derivative Rule for Inverses) If  $f$  has an interval  $I$  as domain and  $f'(x)$  exists and is never zero on  $I$ , then  $f^{-1}$  is differentiable at every point in its domain. The value of  $(f^{-1})'$  at a point  $b = f(a)$  in the domain of  $f^{-1}$  is given by  $(f^{-1})'(b) = \frac{1}{f'(a)}$ .