MATH 161 Extended Syllabus, Fall 2009

Text: Weir, Hass and Giordano: Thomas' Calculus, 11th edition, and
Ziliak, Hulpke: Supplemental notes for Calculus II

Homework problems listed are tentative. **WW** indicates WebWorK, **H** traditional homework to be submitted on paper. These problems are listed in the *last chapter* of the *Supplemental Notes*. **P** marks practice problems from the textbook that cannot be submitted. Practice problems with **N** are from the chapters of the *Supplemental Notes*.

DayWk.DSect.Topic

Aug 24 1 M **First Class**
Prerequisites: College Algebra. Trigonometry. Calculus I
P:Review Calculus I , **WW:**gateway ; Due: Aug 27

Aug 25 T 8.4 **Trigonometric Integrals**
Prerequisites: Trigonometric identities. Derivatives of trigonometric functions. Substitution.
Testable Skills: Integrate any function of the form $\sin^a \cos^b$ for arbitrary a, b; as well as $\sin^a \cos^{-b}$, respectively $\cos^a \sin^{-b}$ for odd a.
P:§8.4: 1-14 , **WW:**sect84 ; Due: Aug 28 , **H:**8.4.A ; Due: Aug 28

A very straightforward chapter. If you encounter a product of arbitrary trigonometric functions you often can write them in terms of sine and cosine. You can ignore examples 4,5 and 6 for the exam.

Aug 26 W 7.1 **Inverse Functions and their Derivatives**
Prerequisites: Functions, Domain and Range of a Function, Solving for x
New Concepts: One-to-One. Enforcing One-to-One by restricting the domain. Inverse Function. Differentiability of the Inverse.
Testable Skills: Give examples of functions which are one-to-one or not. Testing for one-to-one directly. Testing for one-to-one by showing increasing/decreasing. Graph of inverse function. Verify that functions are mutually inverse. Formula for inverse by solving for x. Derivative of the inverse at a point using theorem 1.
P:§7.1: 1-34 , **WW:**sect71 ; Due: Aug 31 , **H:**7.1.A,B ; Due: Sep 4

One of the recurring topics of MATH161 is how to construct new functions. You just need to keep in mind that a function is simply a rule assigning a value, which we call $f(x)$ to every argument value x. We can do so by using a formula, for example $\sin(x^2)$. However we will see that other ways are possible as well.

The method we're using in this chapter is the **inverse**; that is the rule should be the "reverse" of the original rule. You've certainly encountered this concept before: caller ID is the reverse of the phone book. Using your student ID the administration can look up information about you. Important is simply that the information used for lookup is unique; i.e. no two phone (lines) have the same number, or no two students have the same student ID.

The fancy-pants name for this concept is one-to-one. (**Never** write this as 121 or 1-1, as its is very easy to be misread!) For a function f this means that there are no two different x-values, x_1 and x_2, such that $f(x_1) = f(x_2)$. Or, in other words, if we know $f(x)$ there is only one possibility for x. It might not be easy to find this x — it's at least as hard as solving an equation — and there might not even be such an x, but if there is one there is **only** one. This process, finding x if we know $f(x)$, is called the **inverse function**.

There are nice ways of visualizing one-to-one and inverse functions using the graph of the function. However in practice they are of little use: we typically have a function given by a formula or a rule, and plotting the graph, would be hard work. Instead, we want to work with the rule for the function: We show in the lecture that a continuous function which is one-to-one must be strictly increasing or decreasing. That means its derivatives must be positive or negative. This is something we can test without plotting.

Aug 28 F 7.1 **Inverse Functions and their Derivatives (cont.)**

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Aug 31 2 M 7.7 **Inverse Trigonometric Functions**

Prerequisites: Trigonometric Functions. Inverse Functions. Pythagoras' Theorem. Completing the Square.

Testable Skills: Know definition (including domains) of the inverse trigonometric functions for sin, cos, tan, sec. Simplify expressions of the form trig(invtrig(x)). Derivative of the inverse by implicit differentiation. Know derivatives of the inverse trigonometric functions. Integrations that get inverse trigonometric functions in the antiderivatives. Convert expressions of the form $\frac{1}{ax^2+bx+c}$ into $\frac{1}{p}\frac{1}{(x+q)^2+r}$ for integration.

P:§7.7: 1-12, 17-40, 49-112 , **WW:**sect77 ; Due: Sep 3 , **H:**7.7.A-C ; Due: Sep 4

We will not follow the chapter ordering in the book too slavishly but jump a bit. This is no cause for concern (unless you are the textbook author).

There are four main topics in chapter 7.7: First we apply the theory of chapter 7.1 to trigonometric functions. Since these functions are not one-to-one on the whole real axis, we need to restrict their domain. Using theorem one or implicit differentiation, we can calculate derivatives for the inverse trigonometric functions. Then we want to simplify expressions that involve a trigonometric function and the inverse of another trigonometric function, this might look esoteric now, but we will use it later when doing trigonometric substitution.

Finally we will use our knowledge of the derivatives of the inverse trigonometric functions to find antiderivatives of fractions with a quadratic polynomial in the denominator. If this quadratic polynomial does not contain an x-term this is easy: We just need to a linear substitution to transform — for example x^2+a to x^2+1. But in general, if the polynomial contains an x-term, we need to complete the square to transform it to the right form: We write $x^2+ax+b=(x+\frac{a}{2})^2-\frac{a^2}{4}+b$ and then substitute $x+\frac{a}{2}$.

Sep 1 T 7.7 **Inverse Trigonometric Functions (cont.)**

Sep 2 W 7.2 **Natural Logarithms**

Prerequisites: Fundamental Theorem of Calculus. Limit rules.

New Concepts: Natural Logarithm defined as an antiderivative.

Testable Skills: Properties of ln. Derivatives of functions involving ln. Antiderivatives involving ln. Logarithmic differentiation. Relate the logarithm from college algebra to the natural logarithm defined here.

P:§7.2: 1-68 , **WW:**sect72 ; Due: Sep 7 , **H:**7.2.A,B ; Due: Sep 4

You've seen logarithms before in middle school. So why do we do them anew? The definition from school wouldn't let us calculate a derivative. Instead, we define a "new" function which we again call "logarithm". We show that this function has all the properties of the logarithm used in school. Thus we will replace the "old" logarithm with this new one. (We will see the connection to exponentiation in the next chapter.) And as the new logarithm was defined as an integral we know immediately what its derivative is.

The base of this new logarithm might seem to be weird initially; however, as $\log_a(x)=\log_b(x)/\log_b(a)$ logarithms for different bases are simply scaled. The choice of base therefore does not really matter.

In this course, we will **only** consider the natural logarithm, we will never consider logarithms of different bases, such as 2 or 10. (Other bases do not give any new mathematics, they just would confuse things unnecessarily)

x/x^2+1

Sep 4 F 7.3 **Exponentials**

Prerequisites: Natural Logarithm. Inverse Functions.

New Concepts: Exponential function as Inverse of the natural logarithm. Exponential function as its own derivative. Relation of exp to a^b. General definition of exponentiation for arbitrary bases.

Testable Skills: Properties of exp. Derivatives and antiderivatives of a^x and x^a.

P:§7.3: 1-62, 67-69; §7.4: 11-38, 47-70 , **WW:**sect73 ; Due: Sep 7 , **H:**7.3.A ; Due: Sep 11

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Again you might have thought that you learned everything about exponentiation in school. But you don't know what the derivative of 2^x is or how to calculate $3^{\sqrt{5}}$. We therefore define a new function, the inverse of the logarithm, for which we can do calculus using the methods from chapter 7.1. We then see that it behaves like exponentiation of a certain number e by x and therefore define exponentiation in general using this new function: a^b is defined as $e^{b\ln(a)}$. We can thus differentiate and integrate also exponential functions w.r.t other bases, such as 4^x. (This is technically part of chapter 7.4, but there is so little new in this chapter that we do not devote its own lecture to it.)

Sep 7 3 M 7.3 **Exponentials (cont.)**

Sep 8 T **Labor Day (no class)**

Sep 9 W 9.1 **Separable Differential Equations**

Prerequisites: Solving equations. Derivatives. Equality of functions

New Concepts: Differential Equations. General solutions and initial conditions; role of integration constants. Different ways to write differential equations. Separable equations. Implicit and explicit solutions of differential equations. Obstacles: Unknown antiderivatives and impossibility to find explicit solutions.

Testable Skills: Verify that a function fulfills a differential equation. Find suitable parameter values in general solutions to satisfy initial conditions. Determine general solutions to separable differential equations (possibly implicit).

P:§9.1: 1-18 , **WW:**sect91 ; Due: Sep 14 , **H:**9.1.A,B ; Due: Sep 18

Differential equations are a big and important topic and are the reason why your discipline (if it is not mathematics) makes you take calculus. So important in fact, that many of you will take a course, M340, that is devoted to differential equations. We can't cram all of this course in one or two lectures, therefore we just look at a few basic aspects.

> A differential equation is an equation that defines a relationship between a function and one or more derivatives of that function. If only the first derivative arises we talk about a first order differential equation. The general solution of a differential equation is a family of functions, involving parameters. First order equations usually have solutions with only one parameter, we will typically call this parameter C. The values of such parameters are determined by the initial setup of the system. If this is given we call this an initial value problem. By specializing the parameters accordingly we get a particular solution.

You also should be able to recognize a separable equation and know how to solve it. (In general the integration might be too hard or one might not be able to solve an implicit solution for y – you only will have to do examples in which this is easy.)

The following examples of general and particular solutions might be done in class but are not in the book:

DEQ	General solution	Initial values	Particular Solution
$y' + y = \frac{2}{1+4e^{2x}}$	$y(x) = (\operatorname{atan}(2e^x) + C)\,e^{-x}$	$y(-\ln 2) = \frac{\pi}{2}$	$y(x) = e^{-x}\operatorname{atan}(2e^x)$
$\frac{d}{dt}y(t) = e^{-t^2} - 2yt$	$y(t) = (t + C)\,e^{-t^2}$	$y(2) = 0$	$y(t) = (t-2)\,e^{-t^2}$
$x\frac{d}{dx}y(x) + y = -\sin(x)$	$y(x) = \frac{\cos(x)+C}{x}$	$y(\frac{\pi}{2}) = 0$	$y(x) = \frac{\cos(x)}{x}$

Sep 11 F 9.1 **Separable Differential Equations (cont.)**

Sep 14 4 M 7.5 **Exponential Growth and Decay**

Prerequisites: Differential equations. Exponential functions.

New Concepts: Differential equations that lead to exponential growth and their solutions.

Testable Skills: Derive the general solution to an exponential growth problem. Determine growth/decay constant k from given values of a solution. Relation of k to half-life. Determine values at chosen points in time, given the growth constant.

P:§7.5: 1-27 , **WW:**sect75 ; Due: Sep 17 , **H:**7.5.A ; Due: Sep 18

DayWk.DSect.Topic

This is a very straightforward chapter with essentially just four different types of problems:

1. Derive the formula for exponential growth/decay from its differential equation.
2. Given the values at two different times, determine the growth constant k.
3. Given the growth constant k determine the time needed for a specified change (a special case of this is half-life, which is the change to 50

Sep 15 T 7.5 **Exponential Growth and Decay (cont.)**

Sep 16 W **Review**
P:p.547: 1-84,117

For the midterm you should understand the following review questions on p.546/547: 1-5,7,8,11,16-18.
Exam topics are: Inverse Functions, Inverse Trigonometric functions and their derivatives, Logarithm, Exponential function, General solutions to differential equations and initial value problems, exponential growth and decay, and all the integrations related to these topics.

Sep 17 R **Exam 1** — no calculators

Sep 18 F Lab 1**Lab 1: Introduction to Maple**
, **H:**Lab Report ; Due: Sep 25

Meet in Weber 205:

Sep 21 5 M 8.2 **Integration by Parts**
Prerequisites: Product rule.
New Concepts: Integration by parts as converse of the product rule.
Testable Skills: Recognize applicability of integration by parts. Apply integration by parts as step of an integration process. Iterated integration by parts. Treatement of an “invisible one” factor. Integration of $\sin(x)e^x$ via a functional equation.
P:§8.2: 1-30 , **WW:**sect82 ; Due: Sep 24 , **H:**8.2.A ; Due: Sep 25

This is overall a pretty straightforward chapter. If in an exam question you would have to add a factor 1, as in example 3, we will tell you so. You won’t need to know tabular integration or the reduction formula of example 9 for the exam.

Sep 22 T 8.2 **Integration by Parts (cont.)**

Sep 23 W 8.3 **Partial Fractions**
Prerequisites: Fraction arithmetic. Polynomial division. Finding roots of polynomials. Systems of linear equations. Logarithms and inverse trigonometric functions.
New Concepts: Rewriting a rational function as sum of partial fractions.
Testable Skills: Determine the *Ansatz* for partial fractions. Multiply out to obtain equations. Solve these equations to determine the coefficients. Integration for all types of partial fractions. Integration of rational functions.
P:§8.3: 1-40 , **WW:**sect83 ; Due: Sep 28 , **H:**8.3.A,B ; Due: Oct 2

Partial fractions is arguably the most important technique in this course. It can be used for integration, but you will see it again in a course for differential equations and possibly in other contexts. The method doesn’t really use anything beyond basic algebra, but there is a series of steps and calculations sometimes can get a little bit lengthy. An exam question therefore will contain some hints or be only about part of the problem. Still you should know what the steps are and how to do them: first you divide numerator by denominator, then you factor their denominator, then you set up a sum of fractions in which the numerator’s of variables (Know how the original denominator determines the denominators of the summands!), multiply out, and finally solve for the variables. Each summand then will be of a shape which we know how to integrate. As there is limited time, examples in class might not do all steps in detail, you might want to fill in these details at home. You are not required to know the Heaviside method, but if you do you are permitted to use it in an exam.

Sep 25 F 8.3 **Partial Fractions (cont.)**

Sep 28 6 M 8.5 **Trigonometric Substitution**

Prerequisites: Trigonometric functions and trigonometric identities. Simplification of trig(invtrig(x)). Substitution.

New Concepts: Trigonometric substitution as "reverse" substitution. Use of trigonometric substitution for lack of knowledge of a suitable algebraic function.

Testable Skills: Recognize applicability of trigonometric substitution. Understand the kind of trigonometric substitution applicable. Correct substitution (including the dx!) and back substitution. Integration using trigonometric substitution.

P:§8.5: 1-28 , **WW:**sect85 ; Due: Oct 1 , **H:**8.5.A ; Due: Oct 2

Trigonometric substitution looks complicated because it is really a mix of techniques: first we bring a quadratic polynomial in the expression in the form $\pm x^2 \pm a$ by completing the square. Then, depending on the signs, we do one of three substitutions. Note that we do this substitution somehow backwards compared with how we learned substitution originally. We now end up with an integral involving trigonometric functions and we use the techniques from the prior chapters to find an anti-derivative. Then we need to substitute back. In doing so we often encounter a trigonometric function evaluated at the inverse of another trigonometric function. We have seen in chapter 7.7 how to simplify this.

Before you do trigonometric substitution in an exam make sure that you really need this technique and that you cannot solve the integral in an easier way as you could do for example with $\int \frac{x}{x^2+1} dx$.

If you have definite integrals you might find it easier to do trigonometric substitution with an indefinite integral first instead of having to worry about how to treat the limits.

Sep 29 T 8.5 **Trigonometric Substitution (cont.)**

Sep 30 W 8.8 **Improper Integrals**

Prerequisites: Integrals as areas. Limits.

New Concepts: Improper integrals for infinite integration domains and for singularities in the integrand.

Testable Skills: Recognize improper integrals. Evaluate improper integrals as limits.

P:§8.8: 1-34 , **WW:**sect88 ; Due: Oct 5 , **H:**8.8.A ; Due: Oct 9

When we defined integrals we did so for bounded functions on bounded intervals. Sometimes however functions have a peak or we (not just mathematicians but frequently also physicists and engineers) want to integrate to infinity. In either case we do this by considering the integral boundary of variable, doing the definite integration with this variable, and finally taking a limit. Evaluating these limits is a topic on its own which we will not test in an exam. Instead we only look at the question of whether the value of the integral – i.e. the limit – is finite. In some cases we can do this directly, using the limit. In particular the behavior of $\int_0^1 \frac{1}{x^p} dx$ and $\int_1^\infty \frac{1}{x^p} dx$ is important and should be memorized.

Oct 2 F 7.6 **Relative rates of growth**

Prerequisites: Limits, Derivatives

New Concepts: Behaviour at ∞. L'Hospitals rule. Growth order.

Testable Skills: Compare growth order of functions. Given a function, determine a simpler function of same/higher/lower growth order.

P:§7.6: 1-8 , **WW:**sect76 ; Due: Oct 5

We consider to functions to be growing at the same speed if the limit of their quotient is constant. This definition means that adding constants or multiplying by a scalar does not change the growth.

(Consider for example that you want to compare the programs written by two different people on different computers. You want to know what is the better program so simply timing will not work is the one computer might be substantially faster than the other. If – regardless of the size of the input – both programs almost run within a fixed factor of time we consider them equally good.)

The limit calculation to compare growth is typically done using L'Hospital's rule. You might not have seen it, it states that

> If f and g are differentiable functions on $(0, \infty)$ and $\lim_{x\to\infty} f(x) = \infty = \lim_{x\to\infty} g(x)$ then $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{f'(x)}{g'(x)}$.

As a result we get different growth classes, for example polynomials of the same degree.

The only material from this chapter that will appear in exams is to choose suitable functions for comparisons: if you want to show convergence of improper integral you can do so by comparing with the function of the same or faster growth for which the integral still converges; similarly to show a divergence compare with a slower growing function for which the integral still diverges.

Oct 5 7 M **N1** **Convergence of Improper Integrals**

Prerequisites: Improper integrals, Growth

New Concepts: Convergence question for improper integrals Comparison and Limit comparison

Testable Skills: Determine likelyness of convergence/divergence. Determine suitable comparison candidates. Prove convergence/divergence using comparison or limit comparison.

P:§8.8: 37-64; **N**1: 1-10 , **WW:**sect88 ; Due: Oct 8 , **H:N**1: 9,10 ; Due: Oct 9

This chapter is takes from the *Supplemental Notes*. In general, when testing the convergence of improper integrals we don't want t o calculate an anti-derivative (which we would need to actually evaluate the limit) but want to determine convergence with knowledge only of the integrand. The principal method for this is very easy: if we know that the integral for a larger function converges, then also the integral for a smaller function must converge. (Similarly for the case of a smaller function diverging.) And, as the integral of a bounded function in a bounded interval is finite, we need to do this comparison only at infinity for improper integrals of type I. This is the limit comparison test. It also deals with functions being "almost the same". To know what to compare a given function with we will use the growth order as defined in chapter 7.6.

Again, you might not need three full days for 7.6 and this chapter and might want to get to sequences early.

Oct 6 T **N1** **Convergence of Improper Integrals (cont.)**

Oct 7 W 11.1 **Sequences**

Prerequisites: Functions. Continuity. Limits.

New Concepts: Sequences as discrete analogues of functions. Boundedness as condition for convergence. Formal N/ε proofs of convergence.

Testable Skills: Write and interpret formulas for terms of a sequence. Determine the limit of a sequence. Determine N, given ε. Write an N/ε proof for convergence.

P:§11.1, proofs for 23-69. , **WW:**sect111 ; Due: Oct 12 , **H:**11.1.A,B ; Due: Oct 16

A sequence is simply a discrete analogue of a function. You must understand (and know how to apply) the definition of convergence on p. 749. The structure of such a proof is often the same: We set $|a_n - L| < \varepsilon$ and then solve for n. This gives us the needed N-value. We then can write down the proof formally. For example if $a_n = \frac{5n^2+1}{n^2}$ with limit $L = 5$:

First set

$$\varepsilon > |a_n - L| = \left| \frac{5n^2 + 1}{n^2} - 5 \right| = \frac{1}{n^2}$$

and thus $n > 1/\sqrt{\varepsilon}$. Then the proof of the limit looks like this (and this is the kind of writeup, **including text**, we want to see in an exam):

Suppose we have chosen a value of $\varepsilon > 0$. We set $N = \frac{1}{\sqrt{\varepsilon}}$. Then for $n > N$ we have that

$$|a_n - L| = \left| \frac{5n^2 + 1}{n^2} - 5 \right| = \frac{1}{n^2}$$

and (as $n > N$)

$$\frac{1}{n^2} < \frac{1}{N^2} = \frac{1}{(1/\sqrt{\varepsilon})^2} = \varepsilon$$

which by the definition of limit shows that $a_n \to 5$ as $n \to \infty$.

Showing that no limit exists is in general more hairy as one needs to show that no possible L can exist. We will not test this in an exam.

Oct 9 F 11.1 **Sequences (cont.)**

Oct 12 8 M 11.2 **Series**

Prerequisites: Sequences

New Concepts: Sum notation. Divergence criterion. Series as sequences of partial sums.

Testable Skills: Determine partial sums of a series.

P:§11.2: 1-6,23-40,59-62 , **WW:**sect112 ; Due: Oct 15 , **H:**§11.2: 2, 30 ; Due: Oct 16

A series is just a special kind of sequence, namely the sequence of partial sums. We want to study when such series converge (without necessarily calculating the limit). In this case we sum up infinitely many (positive) terms which however sum up to something finite as they get small "quickly enough", in particular the summands must become arbitrarily small. You won't have to know telescoping series.

Oct 13 T 11.2 **Series (cont.)**

Oct 14 W **Review**

P:p.634: 1-126, 135-220 p.547: 97,98 p. 840: 1-18

For the midterm you should understand the following review questions: p.633: 1-4,6-8,10,11,16,17; p.547: 12,13 p.839: 1-3

Exam topics are (in addition to the material from midterm 1 which we assume to be understood): Integration by parts, partial fractions, integrals over powers of sin and cos, trigonometric substitution, improper integrals, comparison and limit comparison for improper integrals, growth order, sequences, convergence proofs of sequences.

Oct 15 R **Exam 2** — no calculators

Oct 16 F Lab 2 **Lab 2: Integration in Maple**

, **H:**Lab Report ; Due: Oct 23

Meet in Weber 205

Oct 19 9 M **N**2 **Geometric Series; Last day to drop with a "W"**

Prerequisites: Series

New Concepts: Reindexing. Geometric Series. Summation formula. Convergence condition for geometric series.

Testable Skills: Reindex a series. Recognize a geometric series. Determine exact values for finite and infinite geometric series.

P:§11.2: 7-14, 41-58,72-77; **N**2: 1-3 , **H:**§11.2: 57,73 ; Due: Oct 23

If there is one topic in the whole course which you are guaranteed to use later in life, regardless what you do, it is the summation formula for the geometric series.

The material for this chapter is from the *Supplemental Notes*.

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Oct 20 T **N**3 **Series as Discrete Analogues of Improper Integrals**

Prerequisites: Riemann sums. Series. Geometric series. Improper integrals. Comparison and Limit comparison.

New Concepts: Correspondence between series and improper integrals. Convergence tests for series. Ratio test.

Testable Skills: Translate improper integrals to series and vice versa. Determine convergence of a series using (limit) comparison tests. Test convergence of a series using a ratio test.

P:§11.3: 1-30; §11.4: 11-36; **N**3: 1-9; §11.5: 1-26 , **WW:**sect1134 ; Due: Oct 23 , **H:N**3: 4,6,10,11 ; Due: Oct 23

Instead of book chapters 11.3 and 11.4 we are using a chapter from the *Supplemental Notes*. The material is essentially a repeat – testing convergence of a series is the same as testing convergence for the corresponding improper integral, and both comparison tests translate.

The ratio test is really just a limit comparison with a suitable geometric series, namely one with ratio lim a_{n+1}/a_n.

Oct 21 W **N**3 **Series as Discrete Analogues of Improper Integrals (cont.)**

Oct 23 F 11.7, **N**4 **Power Series**

Prerequisites: Polynomials. Series. Ratio test. Absolute convergence. Functions.

New Concepts: Power series. Interval as convergence. Functions as power series. Power series as generalizations of polynomials. Power series for rational functions.

Testable Skills: Determine interval of convergence of a power series. Sums and Products of power series. Derivatives and antiderivatives of power series. Verify that a power series fulfills a differential equation. Evaluate power series at certain points. Determine power series for rational functions.

P:§11.7: 1-44, **N**4: 1-3 , **WW:**sect117 ; Due: Oct 28 , **H:**11.7.A,B,C,D ; Due: Oct 30

In this chapter we finally see the reason why we were bothering with series. A power series is a series involving powers of x, thus as long as it converges we obtain a series value for every x-value. In other words: the series describes a function. Such a function might not look as nice as the functions you have seen so far, but in fact has many advantages. By calculating partial sums we can approximate the value of the function. (This is in fact how your calculator evaluates functions such as sine or logarithm. When you press sine there is no little man in the calculator measuring the length among the circle; instead power series is evaluated.)

Power series have an (possibly infinite) interval, centered around the center of the series, in which they are converging (absolute value of ratio < 1), in this course we ignore the behavior at the end points, they converge outside the closed interval.

The Bessel function example and the example of the power series for a rational function are from the *Supplemental Notes*.

Oct 26 10 M 11.7, **N**4 **Power Series (cont.)**

Oct 27 T 11.7, **N**4 **Power Series (cont.)**

Oct 28 W 11.8 **Taylor Series**

Prerequisites: Power series. Differentiation.

New Concepts: Taylor series. Taylor polynomials.

Testable Skills: Calculate taylor polynomials and Taylor series for functions. Know taylor series for e^x, sin, cos.

P:§11.8: 1-31 , **WW:**sect118 ; Due: Nov 3 , **H:**11.8.A,B ; Due: Nov 6

The topic of this chapter and the next is to represent a known function by a power series. You'll see that there is little choice once we have chosen to the center a about which we want to form the power series: it must be the Taylor series of f at $x = a$. (The name Maclaurin series for the special case of a equal zero is not important.)

The partial sums of the Taylor series or called Taylor polynomials.

The main topic of this chapter is how to calculate Taylor polynomials for given functions. Using the formula $c_n = \frac{f^{(n)}(a)}{n!}$ we simply need to evaluate the derivatives of f at a. sometimes we might be able to recognize a general pattern in these coefficients in which lets us write down Taylor polynomials of arbitrary high degree or even a full Taylor series.

For the exam you should be able to calculate Taylor polynomials or Taylor series for functions and to consider such series as power series,I.e. calculate convergence interval, derivative or anti-derivative.

Oct 30 F 11.8 **Taylor Series (cont.)**

Nov 2 11 M 11.9 **More Taylor Series**

Prerequisites: Power Series. Taylor Series.

New Concepts: Modification of power series by algebra, differentiation, antidifferentiation. Identify power series as known functions.

Testable Skills: Know power series for sin, cos, exp atan, $\log(1+x)$. Determine power series/taylor series for more complicated functions from simpler power series. Center shift.

P:§11.9: 1-18, 31-38 , **WW:**sect119 ; Due: Nov 5 , **H:**11.9.A,B ; Due: Nov 6

At this point we study only the second half of this chapter, namely the question of how to get new Taylor series from old ones: As the Taylor series is the only possibility of a power series for a function of the sum of two Taylor series must be the Taylor series for the sum of two functions and so on. This can be very convenient to find new Taylor series. For an exam you should know the Taylor series given in table 11.1 on page 831 and know how to get from this for example a Taylor series for the following functions:

$$\frac{1}{x+1} + \ln(2-x), e^{x/3}, \cos^2(x) = (1+\cos(2x))/2, \frac{x}{(1-x^2)^2}, \sin(x-5)$$

Similarly it is possible to recognize functions from their power series.

We will cover Euler's identity at a later point.

Nov 3 T Lab 3 **Lab 3: Taylor polynomials and Taylor series.**

, **H:**Lab Report ; Due: Nov 10

Meet in Weber 205:

Nov 4 W 11.9 **More Taylor Series (cont.)**

Nov 6 F 11.9, N5**Taylor Series Error estimates**

Prerequisites: Taylor series.

New Concepts: Error term for taylor polynomial approximations. Convergence of Taylor series towards "their" function.

Testable Skills: Estimate the error of a taylor approximation. Determine the necessary degree of a taylor approximation to bound the error.

P:§11.9: 19-30 , **WW:**sect119a ; Due: Nov 10 , **H:**11.9.C,D ; Due: Nov 13

We now cover the main topic of chapter 11.9 which we skipped earlier, namely the question of what the actual value of a Taylor series is. Clearly we would expect it to be the function we started with. However (as you have seen in the lab) it is possible to have perfectly fine functions for which the Taylor series does not converge to function values.

We are investigating whether this happens by considering the error term, that is the difference between the function value and the value of the Taylor polynomial. Taylor's formula gives us an estimate, however it involves the value of the derivative at an intermediate, unknown, point. We therefore estimate even more: we choose a value M which is a bound for the absolute value of the $n+1$-th derivative in the interval. (How do we select M? We look at the derivative and do a very rough estimates for its parts, based on the interval. For example we replace sine by 1, we replace an increasing function by its value on the right boundary and so on.) We then get theorem 23, which is the important result. We will use this theorem in three ways:

1. Given a function and a value for an, estimate how well the degree and Taylor polynomial approximates the function.
2. If we have some bound on the value of M for arbitrary large n, we see that the denominator $(n+1)!$ dominates the formula; the remainder will become arbitrarily small if n gets big. In this situation we can prove that the Taylor series indeed converges towards the function.
3. If a function is a nice and smooth and not to wobbly (which holds for most functions that represent things in the real world) the remainder is small if x is close to a, even for a small n. In this case the Taylor polynomial is a good approximation of the function. This is frequently used in physics and engineering. For example the degree one Taylor polynomial is called linearization; a so called "nonlinear" phenomenon is one in which better than linear approximation of the function is needed to observe the behavior.

Nov 9 12 M 11.9, N5 **Taylor Series Error estimates (cont.)**
Nov 10 T 11.9, N5 **Taylor Series Error estimates (cont.)**

Nov 11 W **Review**
P:p.840: 19-76, p.843, 9-14

For the midterm you should understand the following review questions: p.839: 6-9,11-18,20-28,32

The series in table 11.1 on p. 831 are very important. It is assumed that you know them by heart. Similarly you should know the formulas for the geometric series – both for a partial sum as well as for the infinite limit.

Exam topics are (in addition to the material from midterms 1 and 2 which we assume to be understood): Series and convergence, geometric series, power series, ratio test and convergence interval for power series, arithmetic and calculus for power series, obtaining power series for functions (including deciding on the best method for doing so), identifying power series as functions, Taylor polynomials, expressing unknown antiderivatives by series.

Nov 12 R **Exam 3** — no calculators

Nov 13 F 11.10 **Applications of Power Series**
Prerequisites: Power Series. Taylor Series. Differential equations.
New Concepts: Binomial series. Power series solutions to differential equations.
Testable Skills: Determine power series using the binomial series. Determine power series solutions for differential equations.
P:§11.10: 1-32,57,62-64,69,70 , **WW:**sect1110 ; Due: Nov 16 , **H:**11.10.A ; Due: Nov 20

There are four main topics in this chapter:

1) Binomial Series: You should be able to do problems like example 2 without needing to calculate derivatives. In an exam we would only ask you to do a few coefficients, as evaluating $\binom{n}{k}$ for non-integer n is hard. (Binomial series might be covered in your section only after the midterm, and will in any case only be tested in the final.)
2) Power Series solutions for IVP: Write the solution as power series. Use the initial values to find some coefficients. Use the differential equation to find a recursion (or recursions) for the coefficients. Then solve.
3) Identifying a power series (e.g. a solution of a DEQ) as function: Look at the pattern, you might need to consider the leading terms separately (as added polynomial). We have done this already earlier.
4) Using term-wise integration to find power series for nonelementary integrals.

In the exam we have limited time. Exam problems therefore are likely less involved than some of the homework and might ask only about parts of a problem.

Nov 16 13 M 11.10 **Applications of Power Series (cont.)**

DayWk.DSect.Topic

Nov 17 T 3.5,6.3 **Parametric Curves and Length of Curves**
Prerequisites: Riemann sums
New Concepts: Parametric curves. Length of a curve.
Testable Skills: Understand and sketch parametric curves. Write functions as parametric curves and vice versa. Express the length of a curve as integral.
P:§6.3: 1-3 , **WW:**sect35 ; Due: Nov 20 , **H:**6.3.A ; Due: Nov 20

This is mostly a review of material from M160 which we will need when looking at polar coordinates.

Nov 18 W 3.5,6.3 **Parametric Curves and Length of Curves (cont.)**

Nov 20 F 10.5,10.6 **Polar Coordinates**
Prerequisites: Parametric curves. Trigonometry.
New Concepts: Polar coordinates and polar curves.
Testable Skills: Convert between cartesian and polar coordinates for both points and curves. Determine intersection points of polar curves. Sketch polar curves by hand.
P:§10.5: 1-62,§10.6: 1-16,21-42 , **WW:**sect105 ; Due: Nov 30 , **H:**10.5.A,B; 10.6.A ; Due: Dec 4

This is an easy chapter, and lots of investigation can be done with the calculator. For the exam you should be able to sketch polar curves by hand, find intersection points, and convert between Cartesian and polar. You can ignore the fancy-pants names (such as Limaọn).

Nov 23 **Fall Break**

Nov 30 14 M 10.5,10.6 **Polar Coordinates (cont.)**

Dec 1 T 10.7 **Area and Length in Polar Coordinates**
Prerequisites: Polar coordinates. Length of a curve. Riemann sums.
New Concepts: Formula for area in polar coordinates.
Testable Skills: Determine area in polar coordinates. Use symmetry to simplify area calculations.
P:§10.7: 1-28 , **WW:**sect107 ; Due: Dec 4 , **H:**10.7.A ; Due: Dec 4

As far as technique is concerned, this chapter is rather easy. The formula for length of a curve is really just a special case of the general formula. The resulting integrals can get complicated, and therefore we are rather restricted in what we could ask in an exam.
You should memorize the formula for area in polar coordinates. The only difficulty (when being asked about the area between curves) is finding the intersection points. This formula for area is the first case you see about integration in other coordinates systems. You will encounter this topic again in calculus 3.

You can ignore the surface area section.

Dec 2 W 10.7 **Area and Length in Polar Coordinates (cont.)**

Dec 4 F A.5, 7.8 **Complex Numbers**
Prerequisites:
New Concepts: Complex arithmetic. Absolute value and complex conjugate. Polar ($re^{i\phi}$) form of complex numbers. Complex arithmetic in polar form. Functions and power series over the complex numbers. Relation between exponential and trigonometric functions. Calculus of complex functions. Hyperbolic functions. Complex exponentiation.
Testable Skills: Complex arithmetic in cartesian and polar coordinates. Convert complex numbers between cartesian and polar. Determine complex roots of simple polynomial equations. Evaluate functions and power series over the complex numbers. Calculus using complex numbers.
P:§A.5: 1-24 , **WW:**secta5 ; Due: Dec 9 , **H:**A.5.A,B,C ; Due: Dec 11

For many of you much of this chapter will be repetition. You should know arithmetic, behavior of the complex exponential function and how to find **all** n-th roots of a complex number.

We also introduce the hyperbolic functions for the benefit of other disciplines, but we won't test them in this course.

Dec 7 15 M A.5, 7.8 **Complex Numbers (cont.)**

Day	Wk.D	Sect.	Topic
Dec 8	T	A.5, 7.8	**Complex Numbers (cont.)**
Dec 9	W		**Review** **P:**p.841: 69-80; p.740: 33-68, 77-80

The final is comprehensive. Thus all review for the three midterms is relevant here as well. Exam topics are (in addition to the material from the midterms): Polar coordinates, conversion between cartesian and polar, length of curves and area in polar coordinates, complex numbers in Cartesian and polar form, calculating roots over the complex numbers, exponential and trigonometric functions in polar coordinates.
Corresponding review questions are: p.840: 26,28-32 p.739: 11-14

Dec 11	F		**Review (cont.)**
Dec 14	M		**Final (5.50pm-7.50pm)** — no calculators