M161, Test 2, Spring 2009

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Name: __________________________
Section: _________________________
Instructor: _______________________

Time: 75 minutes. You may not use calculators on this exam

\[
\begin{align*}
\frac{d}{dx} \sin(x) &= \cos(x), & \frac{d}{dx} \cos(x) &= -\sin(x), & \frac{d}{dx} \tan(x) &= \sec^2(x), \\
\frac{d}{dx} \csc(x) &= -\csc(x) \cot(x), & \frac{d}{dx} \sec(x) &= \sec(x) \tan(x), & \frac{d}{dx} \cot(x) &= -\csc^2(x), \\
\frac{d}{dx} \arcsin(x) &= \frac{1}{\sqrt{1-x^2}}, & \frac{d}{dx} \arccos(x) &= -\frac{1}{\sqrt{1-x^2}}, & \frac{d}{dx} \arctan(x) &= \frac{1}{1+x^2}, \\
\frac{d}{dx} \arccsc(x) &= -\frac{1}{x\sqrt{x^2-1}}, & \frac{d}{dx} \arcsec(x) &= \frac{1}{x\sqrt{x^2-1}}, & \frac{d}{dx} \arccot(x) &= -\frac{1}{1+x^2}, \\
\sin(2x) &= 2\sin(x)\cos(x) & \int \ln x \, dx &= x \ln x - x + C & \int \sec(x) \, dx &= \ln |\sec(x) + \tan(x)| + C \\
tan^2(x) + 1 &= \sec^2(x) & \cos^2(x) &= \frac{1 + \cos(2x)}{2} & \sin^2(x) &= \frac{1 - \cos(2x)}{2}
\end{align*}
\]

Multiple Choice Answer Block

A b c d e
B a b c d e
C a b c d e
D a b c d e
E a b c d e
F a b c d e
1) Evaluate the following integrals. Show your work.

a) $\int 1 \cdot \arccos t \, dt$

b) $\int \frac{3x + 2}{x^2(x + 1)} \, dx$
c) \[
\int \frac{1}{(x-1)^2 \sqrt{x^2 - 2x}} \, dx
\]  
(Hint: Start with completing the square under the root.)
2) For each of the following improper integrals determine, using a suitable comparison test, whether the integral converges or diverges. (You do not need to calculate the values of convergent integrals.) State clearly what comparison you are using, what function you compare with, and show that \( f < g \), respectively work out the limit of \( f/g \). No points will be given if no work is shown, or if only an argument of polynomial degrees without explicit comparison is done.

a) \[ \int_{1}^{\infty} \frac{1 + \cos(x)}{x^2} \, dx \]

b) \[ \int_{1}^{\infty} \frac{x \ln x}{x^2 + 2} \, dx \]
The following terms are prototypes of the terms resulting from a partial fraction decomposition. In each case determine an antiderivative:

a) \[ \int \frac{1}{x - 1}\,dx \] (2 points)

b) \[ \int \frac{1}{(x - 1)^2}\,dx \] (2 points)

c) \[ \int \frac{x + 1}{x^2 + 1}\,dx \] (2 points)

d) \[ \int \frac{x + 1}{(x^2 + 1)^2}\,dx \] (10 points)
4) The following multiple choice problems will be graded correct answer only. You do not need to show work, and no partial credit will be given. Record your answer in the answer block on the front page. Answers given on these pages will not be scored. You also may tear off these pages and do not need to hand them in.

It is strongly recommended that you work out the problems until the correct answer is uniquely determined and don’t just try to solve them by “intuition” or “guessing” – doing so is likely to result in a wrong pick.

Each correct answer is worth 6 points, each incorrect answer is counted as 0 points. (Unanswered questions are 1 point, questions in which more than one answer is ticked are considered to have been answered wrongly.)

A) If \( \int f(x) \sin(x) \, dx = -f(x) \cos(x) + \int 3x^2 \cos(x) \, dx \), then \( f(x) \) could be:

\[ a \quad 3x^2 \quad b \quad x^3 \quad c \quad -x^3 \quad d \quad \sin x \quad e \quad \cos x \]

B) Compare the decay of the following functions as \( x \to \infty \):

\[ A : \frac{1}{x^2 \ln(x)} \quad B : \frac{\sqrt{x}}{x^3} \quad C : \frac{x^2}{2x^4 + 3x^3 + 1} \]

\[ a \quad A \text{ decays faster than } B \text{ decays faster than } C \]
\[ b \quad B \text{ decays faster than } A \text{ decays faster than } C \]
\[ c \quad C \text{ decays faster than } A \text{ decays faster than } B \]
\[ d \quad C \text{ decays faster than } B \text{ decays faster than } A \]
\[ e \quad A, B \text{ and } C \text{ decay equally fast} \]
C) The statement \( \lim_{n \to \infty} a_n = L \) means that for each \( \epsilon > 0 \), there exists an \( N \), such that

a) If \( \frac{1}{n} \leq \epsilon \), then \( |a_n - L| < N \)

b) If \( |a_n - L| \leq \epsilon \), then \( n \geq N \)

c) If \( |a_n - a_N| < \epsilon \), then \( L < n \)

d) If \( n \geq N \), then \( |a_n - L| \leq \epsilon \)

e) If \( n \geq N \), then \( |a_n| - |L| \leq \epsilon \)

D) What are all the values \( p \), for which \( \int_{1}^{\infty} \frac{2}{x^{p+1}} dx \) converges?

a) \( p < -1 \)

b) \( p > 0 \)

c) \( p > 1 \)

d) \( p > 2 \)

e) There are no values of \( p \) for which the integral converges.
E) Consider the following three sequences, defined by their terms

\[ a_n = (-1)^n, \quad b_n = (-1)^n/n, \quad c_n = 2^{-n} \]

\[ \text{a} \quad \{a_n\} \text{ and } \{b_n\} \text{ converge; } \{c_n\} \text{ diverges.} \]
\[ \text{b} \quad \{a_n\} \text{ and } \{c_n\} \text{ converge; } \{b_n\} \text{ diverges.} \]
\[ \text{c} \quad \{a_n\} \text{ and } \{c_n\} \text{ diverge; } \{b_n\} \text{ converges.} \]
\[ \text{d} \quad \{a_n\} \text{ diverges, } \{b_n\} \text{ and } \{c_n\} \text{ converge.} \]
\[ \text{e} \quad \{a_n\}, \{b_n\} \text{ and } \{c_n\} \text{ converge.} \]

F) Which of the following improper integrals converge?

\[ A : \int_1^\infty \frac{1}{(x^2 + 1)^2} \, dx \quad B : \int_1^\infty xe^{-x} \, dx \quad C : \int_3^\infty \frac{1}{(2-x)^2} \, dx \]

\[ \text{a} \quad A \text{ only} \quad \text{b} \quad A \text{ and } B \text{ only} \quad \text{c} \quad B \text{ only} \quad \text{d} \quad A \text{ and } C \text{ only} \]
\[ \text{e} \quad B \text{ and } C \text{ only} \]

If you are done and have time left, check your answers on all the problems. Is in each problem clear, what your answer is? Did you tick the correct boxes on the multiple choice questions? Do your calculated antiderivatives differentiate correctly to the original function?