The Derivative Rule for Inverses: If $f$ has an interval $I$ as domain and $f'(x)$ exists and is never zero on $I$, then $f^{-1}$ is differentiable at every point in its domain. The value of $(f^{-1})'$ at a point $b = f(a)$ in the domain of $f^{-1}$ is given by $(f^{-1})'(b) = \frac{1}{f'(a)}$. 

\[
\cos^2 \theta = \frac{1 + \cos 2\theta}{2} \quad \sin^2 \theta = \frac{1 - \cos 2\theta}{2}
\]
1) Let \( f(x) = e^{2x} - 2 \).

a) Give domain and range of \( f \).

b) Explain why \( f \) has an inverse.

c) If possible, find \( f^{-1} \) and give domain and range of \( f^{-1} \).
2) a) If \( \ln(5x) - 2 \ln(y) + \ln(4x) = \ln(A) \), then \( A = ? \)

b) Calculate the derivative \( \frac{d}{dx} \sin(x)\cos(x) \).

c) Simplify \( \tan(\cos^{-1}(x)) \).
3) Evaluate the following integrals. Show your work.

a) \( \int \frac{1}{\sqrt{3x - x^2}} \, dx \)

b) \( \int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx \)
c) \[ \int \frac{1}{x^2 + 10x + 26} \, dx \]

d) \[ \int_{1}^{2} \frac{1}{x^{1.1}} \, dx \]
4) Show that the family of functions $y = Ce^{-2x} + e^{-x}$ are solutions of the differential equation

$$\frac{dy}{dx} + 2y = e^{-x}.$$ 

Also find the value for the constant $C$ to give a solution satisfying the initial value problem $y(-5) = 10$. 
5) A radioactive substance has a half-life of 20 years (that is, after 20 years of decay, only half the amount of the substance remains).

a) Assuming that the decay is governed by the differential equation \( \frac{dy(t)}{dt} = ky(t) \), derive a formula for the amount of substance \( y(t) \) remaining after the time \( t \).

b) Determine the rate of decay \( k \). (As you don’t have a calculator the result will be an expression.)

c) How long does it take for the substance to decay down to \( \frac{1}{10} \) its original amount?