M161, Test 1, Spring 04

NAME: ______________________

SECTION: ____________________

INSTRUCTOR: _________________

You may not use calculators on this exam.

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\[
\cos^2 \theta = \frac{1 + \cos 2\theta}{2} \quad \sin^2 \theta = \frac{1 - \cos 2\theta}{2}
\]
1. (a) Simplify \( \cos \left( \sin \frac{2y}{3} \right) \).

(b) Evaluate \( \cos \frac{\sqrt{3}}{2} \).

(c) Use the definitions of sinh, cosh, tanh, etc to prove

\[
\cosh^2 x = \frac{\cosh(2x) + 1}{2}
\]
2. Calculate the following derivatives (you do not have to simplify).

(a) \( \frac{d}{dx} \sin (2x) \)

(b) \( \frac{d}{dx} [\cosh(3x) + \sinh (\sqrt{2x + 1})] \)

(c) \( \frac{d}{dx} \tan (e^x) \)
3. Evaluate the following integrals. You must show your work.

(a) \( \int \frac{x^2}{x^2 + 1} \, dx \)

(b) \( \int x \ln(x) \, dx \)
(c) \[ \int_{1}^{\infty} \frac{1}{x^{1.001}} \, dx \]

(d) \[ \int \frac{1}{x^2 + 2x} \, dx \]
(e) \[ \int \frac{e^x}{1 + e^{2x}} \, dx \]

(f) \[ \int \frac{16x}{\sqrt{8x^2 + 1}} \, dx \]
4. Derive the formula for integration by parts.
5. Calculate the following limits.

(a) \( \lim_{x \to 0} \frac{x \sin(3x)}{\cos(x) - 1} \)

(b) \( \lim_{x \to \infty} xe^x \)

(c) \( \lim_{x \to 0} \frac{x}{\cos(x)} \)

(d) \( \lim_{x \to 5} \frac{x^2 - 25}{x + 5} \)
6. (a) Which of the functions \( x^2 \) and \( x \ln(x) \) grows faster (or do they grow at the same speed) as \( x \) approaches infinity? Explain.

(b) Which of the functions \( \ln(x) \) and \( \ln(2x) \) grows faster (or do they grow at the same speed) as \( x \) approaches infinity? Explain.
7. Find the solution to the differential equation \( x \frac{dy}{dx} = 4(x^2 + x^2 y^2) \). Write your solution, \( y \), as a function of \( x \).