M161, Final, Fall 2008
Name: ____________________________
Section: ____________________________
Instructor: __________________________

Time: 110 minutes. You may not use calculators on this exam

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Taylor series of the function \( f(x) \) about \( x = a \):
\[
f(a) + \frac{f'(a)}{1!} (x - a) + \frac{f''(a)}{2!} (x - a)^2 + \cdots = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n
\]

\[
\begin{align*}
\frac{d}{dx} \sin(x) &= \frac{1}{\sqrt{1-x^2}}, \\
\frac{d}{dx} \cos(x) &= -\frac{1}{\sqrt{1-x^2}}, \\
\frac{d}{dx} \tan(x) &= \frac{1}{1+x^2}, \\
\int \ln(x) \, dx &= x \ln(x) - x + C \\
\int \sec(x) \, dx &= \ln(\sec(x) + \tan(x)) + C \\
\tan^2(x) + 1 &= \sec^2(x) \\
\cos^2(x) &= \frac{1+\cos(2x)}{2} \\
\sin^2(x) &= \frac{1-\cos(2x)}{2}
\end{align*}
\]

Multiple Choice Answer Block

A | a | b | c | d | e
---|---|---|---|---|---
B | a | b | c | d | e
C | a | b | c | d | e
D | a | b | c | d | e
E | a | b | c | d | e
F | a | b | c | d | e
1) You may write complex numbers in cartesian \((a + bi)\) or polar \((r \cdot e^{i\phi})\) form – either way is accepted.

a) Determine all complex numbers \(z\) such that \(z^5 = -32\).

b) Calculate \(\sum_{n=0}^{\infty} \left(\frac{i}{2}\right)^n\).
2) Evaluate the following integrals. Show your work

a) \( \int \cos(\sqrt{x}) \, dx \) (Hint: Start by substituting \( \sqrt{x} \)).

b) \( \int \frac{9}{(x - 4)^2(x - 1)} \, dx \) (Hint: One of the summands in the partial fractions is \( \frac{-1}{x - 4} \))
c) \[ \int \frac{x^3}{(1 - x^2)^{3/2}} \, dx \]
3) For each of the following improper integrals determine, using a suitable comparison test, whether the integral converges or diverges. (You do not need to calculate the values of convergent integrals.) State clearly what comparison you are using, what function you compare with, and show that $f < g$, respectively work out the limit of $f/g$. No points will be given if no work is shown.

a) $\int_1^\infty \frac{1}{\ln(x^3)} \, dx$

b) $\int_1^\infty \frac{x^3}{e^x} \, dx$
4)  a) Sketch the polar curves $r_1 = 2 + \sin \theta$ and $r_2 = 2 + \cos \theta$ in the following diagram:

![Diagram showing polar curves](image)

b) Determine the points where the curves $r_1 = 2 + \sin \theta$ and $r_2 = 2 + \cos \theta$ intersect.

c) Determine the area inside both curves $r_1 = 2 + \sin \theta$ and $r_2 = 2 + \cos \theta$. (You may use without proof that this area has a symmetry along the line connecting the intersection points.)
5) Determine, for example by appropriate manipulation of known power series, or by calculating the Taylor series about \( a = 0 \), power series for the following functions centered at 0.

Your solution should make clear what steps were performed with series of what functions. If you use “\( + \cdots \)” notation, you should give at least the first four nonzero terms of the series to indicate the pattern of coefficients.

a) \( x \cdot e^{x^2} \)

b) \( \ln(2 + x) = \ln \left( 2 \cdot \left( 1 + \frac{x}{2} \right) \right) \).
6) The following multiple choice problems will be graded correct answer only. You do not need to show work, and no partial credit will be given. Record your answer in the answer block on the front page. Answers given on these pages will not be scored. You also may tear off these pages and do not need to hand them in.

It is strongly recommended that you work out the problems until the correct answer is uniquely determined and don’t just try to solve them by “intuition” or “guessing” – doing so is likely to result in a wrong pick.

Each correct answer is worth 6 points, each incorrect answer is counted as −2 points. (Unanswered questions are 0 points, questions in which more than one answer is ticked are considered to have been answered wrongly.) You cannot get less than 0 points in this part.

A) \( \frac{d}{dx} \ln(e^{2x}) = \)

a) 2x  b) \( \frac{1}{e^{2x}} \)  c) 1  d) 2  e) \( \frac{2}{e^{2x}} \)

B) \( \int_0^1 \frac{x + 1}{x^2 + 2x - 3} \, dx \) is

a) \( \ln \sqrt{3} \)  b) divergent  c) \( -\frac{\ln \sqrt{3}}{2} \)  d) \( \frac{1-\ln \sqrt{3}}{2} \)  e) \( -\ln \sqrt{3} \)
C) After performing integration by parts on $\int 1 \cdot \arcsin x \, dx$ you get

- $\arcsin x + \int \frac{1}{\sqrt{1 - x^2}} \, dx$
- $\sin x - \int \frac{x}{\sqrt{1 - x^2}} \, dx$
- $x \arccos x - \int \frac{x}{\sqrt{1 - x^2}} \, dx$
- $x \arcsin x - \int \frac{x}{\sqrt{1 - x^2}} \, dx$
- $\frac{(\arcsin x)^2}{2} + C$

D) For small values of $h$, the function $\sqrt{16 + h}$ is best approximated by which of the following?

- $h/32$
- $4 + h/32$
- $4 - h/32$
- $2 - h/32$
- $2 + h/32$
E) The length of the curve $y = \ln \sec x$ from $x = 0$ to $x = \frac{\pi}{4}$ may be expressed by which of the following integrals?

- $\int_{0}^{\frac{\pi}{4}} \sec x \tan x \, dx$
- $\int_{0}^{\frac{\pi}{4}} \sec x \, dx$
- $\int_{0}^{\frac{\pi}{4}} \sqrt{1 + (\ln \sec x)^2} \, dx$
- $\int_{0}^{\frac{\pi}{4}} \sqrt{1 + (\sec^2 x \tan^2 x)} \, dx$
- $\int_{0}^{\frac{\pi}{4}} \sec^2 x \, dx$

F) Suppose that $f$ is a function that is defined for all real numbers. Which of the following conditions assures that $f$ has an inverse function?

- The graph of $f$ is concave up.
- The function $f$ is periodic.
- The function $f$ is a strictly increasing function.
- The function $f$ is continuous.
- The graph of $f$ is symmetric with respect to the $y$-axis.