

YOUR TITLE GOES HERE

YOUR NAME GOES HERE

1. EXISTENCE

Before attempting to solve a differential equation, it would be nice to know whether or not a solution exists at all. To prove existence of solutions we use the method of successive approximations (developed by French mathematician Emile Picard (1856-1941)). The idea is based on the following observation: the function $y(t)$ solves the initial value problem

$$(1.1) \quad y' = f(t, y), \quad y(a) = b$$

on some interval I containing $t = a$ if and only if it satisfies the integral equation

$$(1.2) \quad y(t) = b + \int_a^t f(s, y(s)) ds$$

for all $t \in I$.

We attempt to solve the integral equation (1.2) iteratively. That is, we iteratively define a sequence $y_1, y_2, \dots, y_n, \dots$ of functions that we hope will converge to the solution. We start with

$$y_0(t) = b$$

and let

$$y_1(t) = b + \int_a^t f(s, y_0(s)) ds,$$

and having defined the first n functions y_0, y_1, \dots, y_{n-1} , we let

$$(1.3) \quad y_n(t) = b + \int_a^t f(s, y_{n-1}(s)) ds.$$

Suppose that each of the functions in the sequence $\{y_n\}_{n=0}^{\infty}$ is defined on some open interval (the same for each n) containing $t = a$, and that the limit $y(t) = \lim_{n \rightarrow \infty} y_n(t)$ exists at each point of this interval. If the convergence, $y_n(t) \rightarrow y(t)$ is uniform and $f(x, z)$ is continuous in the z variable, then we can obtain the

following:

$$\begin{aligned}
 y(t) &= \lim_{n \rightarrow \infty} y_n(t) \\
 &= \lim_{n \rightarrow \infty} \left[b + \int_a^t f(s, y_{n-1}(s)) ds \right] \\
 &= \lim_{n \rightarrow \infty} b + \lim_{n \rightarrow \infty} \int_a^t f(s, y_{n-1}(s)) ds \\
 &= b + \int_a^t f(s, \lim_{n \rightarrow \infty} y_{n-1}(s)) ds \\
 &= b + \int_a^t f(s, y(s)) ds.
 \end{aligned}$$

2. PICTURES

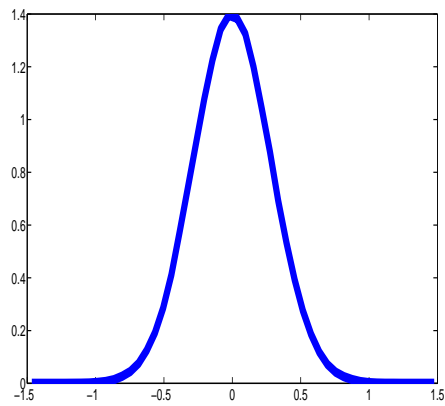


FIGURE 1. KDE of q

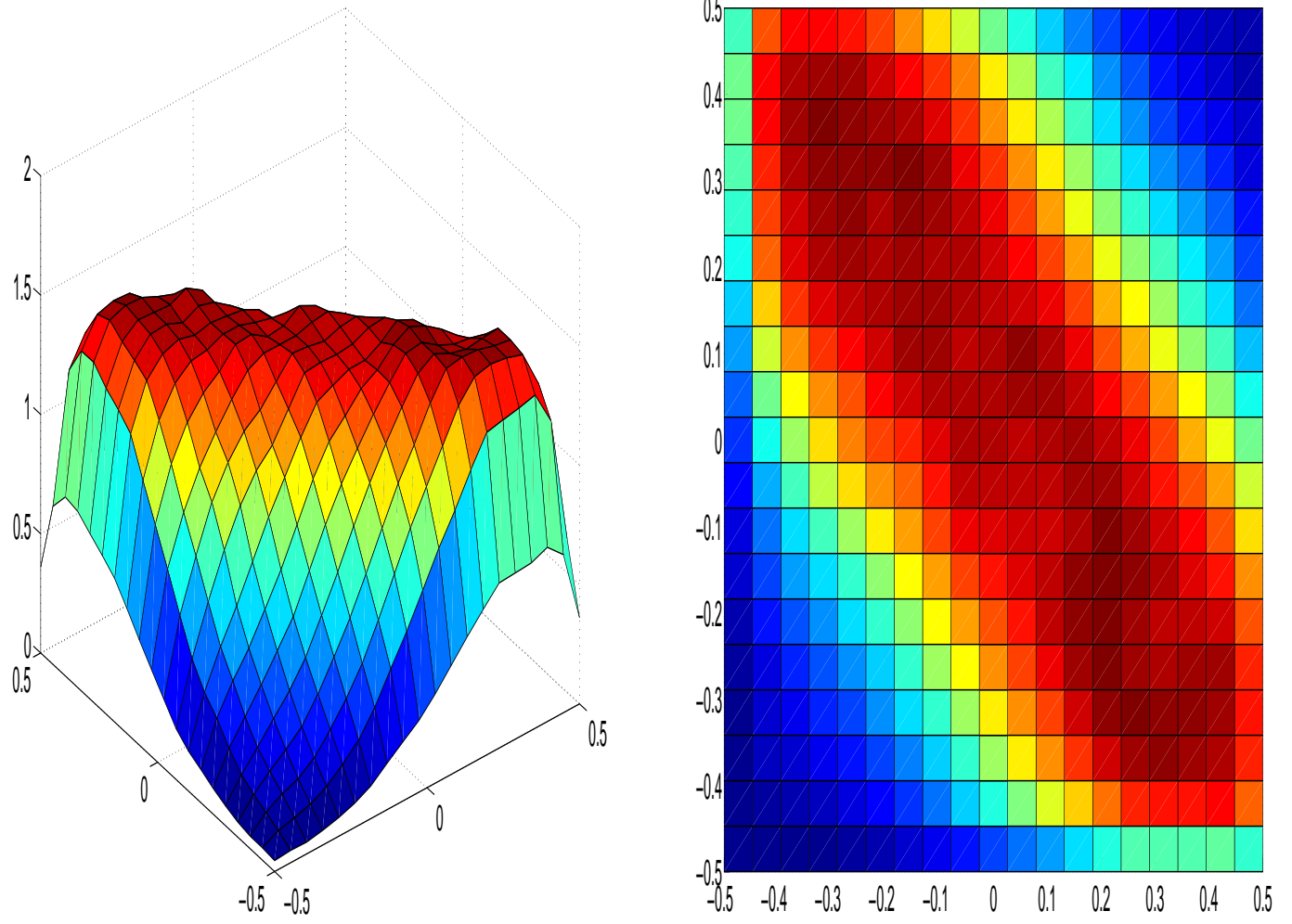


FIGURE 2. KDE of joint distribution of parameters sampled w.r.t density $\rho_{m,1}(\theta)$

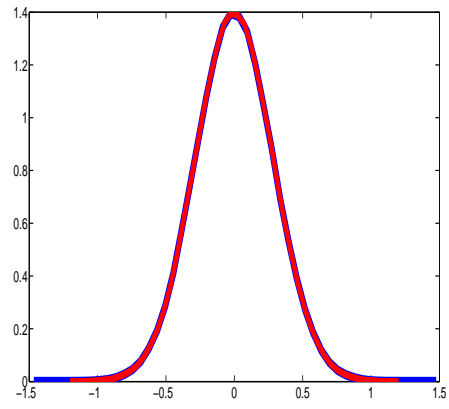


FIGURE 3. KDE of q w.r.t. Normally distributed parameters (blue) and parameters from $\rho_{m,1}(\theta)$ (red)