Simple 6- and 7-designs on 19 to 33 points

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Abstract

Recent results in the search for simple t-designs are reported. There are 31 parameter sets of simple 7-designs and many parameter sets of new simple 6-designs on up to 33 points listed up. The tool used is a program DISCRETA, developed by the authors, which applies the method of Kramer-Mesner [6] where an automorphism group of the desired designs is prescribed. If the automorphism group is large enough group theoretical arguments allow to determine the number of isomorphism types of designs found. The search was successful with prescribed projective linear groups and some extensions of cyclic groups. In several cases Tran van Trung's construction [17] yields further results from those found by computer.

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1 Introduction

A simple t-(v, k, λ) design $\mathcal{D}$ is defined as a set of k-subsets, called blocks, of a set V of v points such that each t-subset of V is contained in the same number λ of blocks. Since we only consider simple designs, we omit the word simple. For a long time t-designs were known only for $t \leq 5$. Then, in 1984, S. Magliveras and D. W. Leavitt [10] constructed the first 6-designs using the method of Kramer-Mesner [6] and a computer. As a big sensation, L. Teirlinck [15] in 1989 proved that t-$(v, t+1, \lambda)$ designs exist for all t. The proof is constructive, but the resulting designs, namely $t-(v, t+1, \lambda)$ designs, have astronomically large parameters v and λ like $\lambda = (t+1)!^{2t+1}$ and $v \equiv t \mod \lambda$. Thus, small examples and cases where k is greater than $t+1$ are still interesting. Some small 6-designs and even two infinite series of 6-designs had been found. A recent review can be found in the Handbook of Combinatorial Designs [8]. Most designs with small parameter sets and "large" t were found using Kramer-Mesner matrices. We also follow this approach and, like D. L. Kreher and S. P. Radziszowski [9], use an LLL-algorithm for solving systems of diophantine equations [18]. Our program system DISCRETA allows to choose a permutation group from several series like projective linear groups and make some group constructions. The group thus determined is then prescribed as an automorphism group of the desired designs. Any such design is a collection of full orbits of that group on the set of

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k-subsets of V. Finding a collection which forms a t-design is equivalent to the problem of solving a system of diophantine equations, as mentioned above.

The most successful choices of permutation groups in our search are projective linear groups. All groups $\mathrm{PSL}(2, q)$ for q a prime power between 19 and 32 are admitted as automorphism groups of some 6-design. In particular this search completes the work started by [5] on $\mathrm{PSL}(2, 19)$ and $6 \leq t \leq 8$ and $k = 10$. We remark that no 8-$(20, 10, \lambda)$ design with this automorphism group exists. The case $k = 9$, $t = 8$ admits no feasible parameter set. In addition, a 6-$(19, 7, 4)$ design and a 6-$(19, 7, 6)$ design have been found by prescribing the groups $Hol(C_{17}) + +$, and $Hol(C_{19})$, respectively, where the $+$ operator adds a fixed point to a permutation group. There are only two smaller 6-designs known, the parameters are 6-$(14, 7, 4)$ [9] and their automorphism group is $C_{13}+$. These groups are large enough such that group theoretical arguments allow to determine the number of isomorphism types of designs admitting one such group as a group of automorphisms. This requires to find all 0/1-solutions of the diophantine equations which is done for the smaller cases.

Of course, designs with even higher t would be a good challenge. Therefore, one tries to extend existing t-designs to $(t+1)$-designs. This works only in very rare cases. It is easy to get t-designs from a $(t+1)$-design, the three standard constructions are to form the residual design, the derived design and to consider the $(t+1)$-design as a t-design. The extension process would just reverse these three processes. We interpret a theorem by Tran van Trung [17] from this point of view and see that from two t-designs with the parameters of a derived and a residual design one obtains the third design derivable from the possibly existing $(t+1)$-design. Thus, each parameter set for which a design has been found gives rise to the question whether one of its buddies needed for Tran van Trung's construction also has a design. The existence of such a design is necessary for the existence of an extended design, but it is not yet sufficient. Nevertheless it is sufficient to obtain the third partner with the same t. We took this observation as a guideline to enrich the catalogue of 7- and 6-designs with good success. It is only a pity that we could not yet establish any 8-design in this way. The parameter sets of designs found so far are listed below.

2 Methods

We shortly review some well known combinatorial properties of t-designs. Let $\mathcal{D}$ be a t-(v, k, λ) design defined on a point set V. Then we can form some new designs from $\mathcal{D}$. Let S be a s-subset of V, where $s \leq t$. Counting twice all pairs (T, B) such that $S \subset T$ and B is a block of $\mathcal{D}$ yields

$$\lambda_s = \lambda \binom{v-s}{t-s} / \binom{k-s}{t-s}.$$

Thus, $\mathcal{D}$ is also a s-(v, k, λ_s) design for each such s. Usually, if we speak of the parameters of a design $\mathcal{D}$ the largest known value of t for which $\mathcal{D}$ is a t-design is meant. The special value $s = t - 1$ is a recursion formula

$$\lambda_{t-1} = \lambda_t \frac{v - t + 1}{k - t + 1}.$$

Distinguishing a point $x \in V$ separates the blocks of $\mathcal{D}$ into those containing x and those not containing x. Removing x from V then turns both classes of blocks into $(t-1)$-designs. The blocks which beforehand contained x afterwards have size $k - 1$ and form a $(t-1)$-$(v-1, k-1, \lambda)$ design, known as the derived design $der_x(\mathcal{D})$. The other blocks form a $(t-1)$-$(v-1, k, \lambda_{(t-1)} - \lambda_t)$ design, called the residual design $res_x(\mathcal{D})$. While the isomorphism types of the residual and derived designs of a design may depend on the special choice of the point x taken from V, the parameter sets are independent of this choice.

We consider parameter sets independently from the designs and define the following operations on them, which correspond to the above constructions of designs:

- $red: \ t\text{-}(v, k, \lambda) \ \mapsto \ (t-1)\text{-}(v, k, \lambda_{(t-1)})$
- $der: \ t\text{-}(v, k, \lambda) \ \mapsto \ (t-1)\text{-}(v-1, k-1, \lambda)$
- $res: \ t\text{-}(v, k, \lambda) \ \mapsto \ (t-1)\text{-}(v-1, k, \lambda_{(t-1)} - \lambda_t)$.

These operations all can be applied doing simple arithmetic. It is easy to see that they commute pairwise. So, starting from one parameter set one obtains several new parameter sets. Generally, there may result non-integer fractions for some λ_s. Such a parameter set cannot belong to a design. Clearly also all parameter sets are ruled out which led to this case. A parameter set is called *admissible* if all operations result in integer valued parameter sets when applied in any combination.

It is easy to compute preimages, but they also may contain rational non-integer values. The most interesting question is whether in case of an integer valued preimage parameter set a corresponding design exists. This is not easy to answer but a weaker result holds.

Theorem (Tran van Trung's construction) [17]: *Let $PS = t$-(v, k, λ) be a set of admissible parameters such that there exist designs for the parameter sets der(PS) and res(PS). Then there exists a design with the parameter set red(PS).*

The above process to obtain a residual design relies on the observation that there are exactly $\lambda_{(t-1)} - \lambda_t$ blocks containing a given $(t-1)$-set and not containing a fixed point x. Remarkably, this number is independent of the choice of x. This can be generalized to the following result.

Theorem [12]: *For a pair (S, J) such that S and J are disjoint and $|S| + |J| \leq t$ let $\lambda_{|S|}^{|J|}$ be the number of blocks containing S and disjoint from J. Then this number is independent of the choice of the pair.*

Proof The proof is by induction on $j = |J|$. For $j = 0$ we have $\lambda_i^0 = \lambda_i$ for all i. Then the recursive formula

$$\lambda_i^{(j+1)} = \lambda_i^j - \lambda_{i+1}^j$$

gives the values for larger j. Obviously these are independent of the choice of S. $\diamond$

These intersection numbers appear as parameter values of λ when Tran van Trung's construction can be applied in more than one step, since derived designs of derived designs appear. We will show such cases below. A general formulation is contained in Tran van Trung's paper [17].

To start the combinatorial machinery of design constructions we described so far one clearly needs a substantial basic set of designs. We use prescribed automorphism groups to construct designs. This is the Kramer-Mesner method [6].

We have built a software package DISCRETA with a graphical interface for this method. We shortly list some features of the system.

- A group can be chosen by pressing a button for a certain family of groups and adding some special parameter values to pick a specific group out of this family. We mention the families $\mathrm{PSL}(n,q), \mathrm{PGL}(n,q), \mathrm{P\Sigma L}(n,q), \mathrm{P\Gamma L}(n,q)$, of projective linear groups, cyclic groups and their holomorphs, induced actions of symmetric groups. In addition, one can read in permutations generating the desired group from a file or give a set of generators and defining relations for the group and apply a module for low index methods to get the permutation representation needed.

- One can manipulate these groups by forming sums, products, wreath products, by adding fixed points or forming stabilizers. So, if a group is chosen the point set together with the corresponding permutation representation is automatically determined.

- Now the design parameters t and k can be set. A button allows to find out which values of λ are allowed for a feasible parameter set.

- If the set of parameters looks promising one can start the computation of the Kramer-Mesner matrix by pressing the appropriate button. Internally, double coset algorithms are used to construct that matrix. The program is a new implementation of Schmalz's Leiterspiel [14] by A. Betten who also wrote most of the code of the system.

- One may look at the result to see whether some nice properties are immediate. So, one may detect a column which has constant entries. This column then corresponds to a design where the given group of automorphism is transitive on the set of blocks. Also, the existence or non-existence of designs with small λ (small Steiner systems for instance) is easily recognized.
- To get more complicated combinations of columns which together form a t-design we have two special buttons. A program written by B. D. McKay performs a clever backtrack search and is especially useful in showing that no design with the prescribed parameter values and group exists. Also, at least for smaller values of λ or matrices with only few rows and many columns this program is our best choice. Alternatively, a program by A. Wassermann with his version of the LLL-algorithm solves the systems of equations even in cases of some hundred rows and columns [18]. Interestingly, this version sometimes finds designs also for values of λ different from the one given as input to DISCRETA. This is remarkable because it may occur that because of the computational complexity these solutions are only very hard to obtain directly by giving as input these values of λ which are found by chance.
- A database is used for keeping the parameter sets found. This allows to ask for all stored parameter sets where each parameter lies in a given range. Inserting a new parameter set PS automatically generates the parameter sets der(PS), red(PS), and res(PS) recursively and stores them in the database. This way, a lot of parameter sets are easily generated from a few starting parameter sets. In this text only the original new parameter sets of the designs are included.

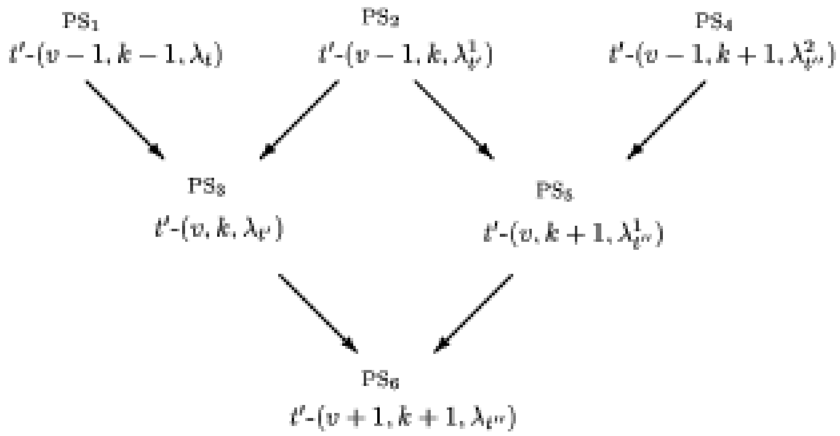
We found that the graphical surface was very helpful for testing ideas immediately. Since the package is built from independent modules, it is not too difficult to integrate new features. So, further features are under development.

In our experience, the LLL-module is very powerful and allows to get all solutions when we are patient enough. In general, different solutions may be isomorphic designs. We mention shortly, how we handle this problem without isomorphism testing in our cases. For a detailed exposition of this approach see Schmalz [14] for the first part and furthermore [3]. If the prescribed group is a maximal subgroup of the symmetric group on the point set then all solutions are pairwise nonisomorphic. More generally, if the given group is the full automorphism group then only the normalizer taken in the symmetric group may map one solution onto another. In this case the index of the group in its normalizer is the number of solutions belonging to the same isomorphism type of designs. Since the full automorphism group is at least as big as the given group one has to subtract from the solutions first all those which are also solutions of a proper overgroup. This is most easily done when there are only few overgroups and only the number of solutions is desired. A different method is applicable if the given group contains a Sylow subgroup of the symmetric group. Then it suffices to look at elements of the normalizer of that Sylow subgroup to find permutations mapping one design admitting the given group onto another such design [3]. This latter method especially applies to holomorphs of cyclic groups of prime order. The number of isomorphism types given in the table below are determined in this way.

3 Results

We searched for 6-designs and 7-designs on up to 33 points. A few parameter sets in this range were already known, see D. Kreher's contribution in the Handbook of Combinatorial Designs [8]. We include them into our report below, since in some cases we could add information on automorphisms and isomorphism types.

We used the Tran van Trung construction several times to get new 6- and 7-designs from existing ones. The most general scheme with an iterated application we found is as follows, using $t' = t - 1$, $t'' = t - 2$ in the notation from above:



In this scheme PS_3 is obtained from PS_1 and PS_2, PS_5 from PS_2 and PS_4, and PS_6 from PS_3 and PS_5, by Tran van Trung's construction.

This scheme appears for $PS_1 \in \{7\text{-}(24, 8, 6), 7\text{-}(24, 8, 8)\}$. With PS_4 missing the scheme appears for $PS_1 \in \{6\text{-}(29, 8, \lambda) | \lambda = 36, 42, 64, 85, 99, 105, 112, 126\}$. The smallest scheme with only PS_1, PS_2, PS_3 appears for $PS_1 \in \{7\text{-}(24, 8, 5), 7\text{-}(26, 8, 6), 6\text{-}(29, 8, 120)\}$.

In addition there are several isolated results. The full list of parameter sets known to us together with automorphism groups and number of isomorphism types in smaller cases is shown in the following table. In some cases an exact number of isomorphism types is given. Then this is the number of isomorphism types of designs with the prescribed automorphism group. If no number is given only the existence of designs with that parameter set and, in some cases, the indicated automorphism group is asserted. Many isomorphism types means hundreds and mostly thousands of isomorphism types which were not fully determined because of the amount of computer resources that would have been needed. The Tran van Trung construction is abbreviated as TvT. For this construction no automorphism groups and no number of isomorphism types is given. The table does not list residual, derived, complementary, supplementary designs and s-designs for $s \leq t$. Details on the designs mentioned and not cited from the literature can be obtained from the authors, see also our WWW-page:

http://www.mathe2.uni-bayreuth.de/betten/DESIGN/d1.html

parameter set	constructed by	isomorphism types
7-(33, 8, 10)	$P\Gamma L(2,32)$	4996426 [2], [18]
7-(30, 9, 105)	$P\Gamma L(2,27)++$	
7-(30, 9, 112)	$P\Gamma L(2,27)++$	many
7-(28, 10, 630)	$P\Gamma L(2,27)$	many
7-(27, 9, 60)	TvT: 7-(26,8,6) ∪ 7-(26,9,54)	[4], [3]
7-(27, 10, 240)	$P\Gamma L(2,25)+$	≥ 1
7-(27, 10, 540)	$P\Gamma L(2,25)+$	many
7-(26, 8, 6)	PGL(2,25)	7 [4], [3]
7-(26, 9, 81)	$P\Gamma L(2,25)$	many [4], [3]
7-(26, 9, 63)	$P\Gamma L(2,25)$	37932 [4], [3]
7-(26, 9, 54)	$P\Gamma L(2,25)$	3989 [4], [3]
7-(26, 10, 342)	TvT: 7-(25,9,54) ∪ 7-(25,10,288)	
7-(26, 10, 456)	TvT: 7-(25,9,72) ∪ 7-(25,10,384)	
7-(25, 9, 45)	TvT: 7-(24,8,5) ∪ 7-(24,9,40)	[4], [3]
7-(25, 9, 54)	TvT: 7-(24,8,6) ∪ 7-(24,9,48)	[4], [3]
7-(25, 9, 72)	TvT: 7-(24,8,8) ∪ 7-(24,9,64)	[4], [3]
7-(25, 10, 288)	TvT: 7-(24,9,48) ∪ 7-(24,10,240)	
7-(25, 10, 384)	TvT: 7-(24,9,64) ∪ 7-(24,10,320)	
7-(24, 8, 4)	PSL(2,23)	1 [4], [3]
7-(24, 8, 5)	PSL(2,23)	138 [4], [3]
7-(24, 8, 6)	PSL(2,23)	$\geq$ 132 [4], [3]
7-(24, 8, 7)	PSL(2,23)	$\geq$ 126 [4], [3]
7-(24, 8, 8)	PSL(2,23)	$\geq$ 63[4], [3]
7-(24, 8, 8)	PGL(2,23)	4
7-(24, 9, 40)	PGL(2,23)	113 [4], [3]
7-(24, 9, 48)	PGL(2,23)	$\geq$ 2827[4], [3]
7-(24, 9, 64)	PGL(2,23)	$\geq$ 15335 [4], [3]

parameter set	constructed by	isomorphism types
7-(24, 10, 240)	PGL(2,23)	
7-(24, 10, 320)	PGL(2,23)	$\geq$ 2
7-(20, 10, 116)	PSL(2,19)	3
7-(20, 10, 124)	PSL(2,19)	1
7-(20, 10, 134)	PSL(2,19)	10
6-(33, 8, 36)	$P\Gamma L(2,32)$	1179 [5], [14]
6-(32, 7, 6)	$PSL(2,31)$	$\geq$ 18
6-(31, 10, 1800)	TvT: 6-(30,9,288) $\cup$ 6-(30,10,1512)	
6-(31, 10, 2100)	TvT: 6-(30,9,336) $\cup$ 6-(30,10,1764)	
6-(31, 10, 3200)	TvT: 6-(30,9,512) $\cup$ 6-(30,10,2688)	
6-(31, 10, 4250)	TvT: 6-(30,9,680) $\cup$ 6-(30,10,3570)	
6-(31, 10, 4950)	TvT: 6-(30,9,792) $\cup$ 6-(30,10,4158)	
6-(31, 10, 5250)	TvT: 6-(30,9,840) $\cup$ 6-(30,10,4410)	
6-(31, 10, 5600)	TvT: 6-(30,9,896) $\cup$ 6-(30,10,4704)	
6-(31, 10, 6300)	TvT: 6-(30,9,1008) $\cup$ 6-(30,10,5292)	
6-(30, 7, 12)	$PSL(2,29)$, 6-(8m+6,7,4m) for m=3 [16]	many
6-(30, 9, 288)	TvT: 6-(29,8,36) $\cup$ 6-(29,9,252)	
6-(30, 9, 336)	TvT: 6-(29,8,42) $\cup$ 6-(29,9,294)	
6-(30, 9, 512)	TvT: 6-(29,8,64) $\cup$ 6-(29,9,448)	
6-(30, 9, 680)	TvT: 6-(29,8,85) $\cup$ 6-(29,9,595)	
6-(30, 9, 792)	TvT: 6-(29,8,99) $\cup$ 6-(29,9,693)	
6-(30, 9, 840)	TvT: 6-(29,8,105) $\cup$ 6-(29,9,735)	
6-(30, 9, 896)	TvT: 6-(29,8,112) $\cup$ 6-(29,9,784)	
6-(30, 9, 960)	TvT: 6-(29,8,120) $\cup$ 6-(29,9,840)	
6-(30, 9, 1008)	TvT: 6-(29,8,126) $\cup$ 6-(29,9,882)	
6-(30, 10, 1512)	$P\Gamma L(2,27)++$	$\geq$ 168
6-(30, 10, 1764)	$P\Gamma L(2,27)++$	many
6-(30, 10, 2688)	$P\Gamma L(2,27)++$	many
6-(30, 10, 3570)	$P\Gamma L(2,27)++$	many
6-(30, 10, 4158)	$P\Gamma L(2,27)++$	$\geq$ 404
6-(30, 10, 4410)	$P\Gamma L(2,27)++$	$\geq$ 8300
6-(30, 10, 4704)	$P\Gamma L(2,27)++$	many
6-(30, 10, 4914)	$P\Gamma L(2,27)++$	
6-(30, 10, 4956)	$P\Gamma L(2,27)++$	
6-(30, 10, 5082)	$P\Gamma L(2,27)++$	
6-(30, 10, 5166)	$P\Gamma L(2,27)++$	
6-(30, 10, 5292)	$P\Gamma L(2,27)++$	$\geq$ 4970
6-(29, 8, 36)	$P\Gamma L(2,27)+$	8

parameter set	constructed by	isomorphism types
6-(29, 8, 42)	$P\Gamma L(2,27)+$	31
6-(29, 8, 43)	$P\Gamma L(2,27)+$	43
6-(29, 8, 49)	$P\Gamma L(2,27)+$	479
6-(29, 8, 57)	$P\Gamma L(2,27)+$	5177
6-(29, 8, 63)	$P\Gamma L(2,27)+$	17195
6-(29, 8, λ), λ = 64,70,78,84,85,91,99, 105,106,112,120,126	$P\Gamma L(2,27)+$	many
6-(29, 9, λ), λ = 105,126,154,252,294, 322,406,448,469,483, 504,532,595,630,672, 693,735,756,784,798, 819,826,840,861,882	$P\Gamma L(2,27)+$	many
6-(29, 10, 4095)	$P\Gamma L(2,27)+$	
6-(29, 10, 4305)	$P\Gamma L(2,27)+$	
6-(28, 8, 42)	$P\Gamma L(2,27)$	2 [14]
6-(28, 8, 63)	$P\Gamma L(2,27)$	367 [14]
6-(28, 8, 84)	$P\Gamma L(2,27)$	21743 [14]
6-(28, 8, 105)	$P\Gamma L(2,27)$	38277 [14]
6-(26, 8, 60)	$P\Gamma L(2,25)$	23
6-(26, 8, 70)	$P\Gamma L(2,25)$	87
6-(25, 8, 36)	PGL(2,23)+	242 [4]
6-(25, 8, 45)	PGL(2,23)+	10008 [4]
6-(25, 8, 54)	PGL(2,23)+	[4]
6-(25, 8, 63)	PGL(2,23)+	1284 [4]
6-(25, 8, 72)	PGL(2,23)+	[4]
6-(25, 8, 81)	PGL(2,23)+	[4]
6-(24, 8, 36)	PGL(2,23)	9[4]
6-(24, 8, 45)	PGL(2,23)	49 [4]
6-(24, 8, 54)	PGL(2,23)	476[4]
6-(24, 8, 63)	PGL(2,23)	1284[4]
6-(24, 8, 72)	PGL(2,23)	3069 [4]
6-(23, 8, 68)	[7]	
6-(22, 8, 60)	PSL(2,19)++	1148 [7]
6-(22, 7, 8)	6-(8m+6,7,4m) for m=2 [16]	
6-(20, 9, 112)	PSL(2,19)	2 [5]
6-(19, 7, 4)	$Hol(C_{17}) + +$	1
6-(19, 7, 6)	$Hol(C_{19})$	3
6-(14, 7, 4)	$C_{13}+$	2 [9]

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