Simple 7-Designs With Small Parameters

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ABSTRACT

We describe a computer search for the construction of simple designs with prescribed automorphism groups. Using our program package DISCRETA this search yields designs with parameter sets 7-(33, 8, 10), 7-(27, 9, 60), 7-(26, 9, λ) for λ = 54, 63, 81, 7-(26, 8, 6), 7-(25, 9, λ) for λ = 45, 54, 72, 7-(24, 9, λ) for λ = 40, 48, 64, 7-(24, 8, λ) for λ = 4, 5, 6, 7, 8, 6-(25, 8, λ) for λ = 36, 45, 54, 63, 72, 81, 6-(24, 8, λ) for λ = 36, 45, 54, 63, 72, 5-(19, 6, 4), and 5-(19, 6, 6). In several of these cases we are able to determine the exact number of isomorphism types of designs with that prescribed automorphism group. © 199? John Wiley & Sons, Inc.

Keywords: *t-design, Kramer-Mesner method, isomorphism problem*

1. INTRODUCTION

Simple t-(v, k, λ) designs which are constructed via large sets tend to have large parameters, at least for $t > 5$ [25]. In 1984, S. S. Magliveras and D. W. Leavitt [20] presented the first simple 6-designs which were found using a prescribed group of automorphisms. They employed a method of Kramer and Mesner [13] which constructs the set of blocks of the design as a union of orbits of a prescribed group on sets. In contrast to the method of large sets, the designs found with a prescribed group tend to have small parameters. Since then, several other 6-designs have been found the Kramer-Mesner method. In many cases also the number of isomorphism types of designs with prescribed automorphism group could be deter-

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mined [17, 16, 18, 4, 21, 22]. A survey on the search for t-designs with small v is contained in the article of D. L. Kreher in the CRC-Handbook of Combinatorial Designs [5]. Further results are reported by the first author on his homepage **http://www.mathe2.uni-bayreuth.de/betten/DESIGN/d1.html**. Using some refined methods for constructing Kramer-Mesner matrices and solving large systems of Diophantine linear equations, 7-designs with parameters 7-(33, 8, 10) and prescribed group of automorphisms $P\Gamma L(2,32)$ could be found by the end of 1994 [3]. At that time, B. D. McKay noticed that there exist thousands of such designs and estimated a total of about 5 million designs of this type. Meanwhile, the third author could completely settle the existence question using refined techniques for enumerating designs (cf. [27]). He was able to enumerate all 4 996 426 designs of that type, which is surprisingly close to the estimated number. The full set of designs can be obtained electronically via Internet from our homepage for this article (see below).

In this article, we show that simple 7-designs exist for even smaller parameters.

Theorem 1.1. *There exist exactly 7 isomorphism types of simple* 7-(26, 8, 6) *designs with automorphism group* $PGL(2,25)$. *There exist exactly* 3989 *and* 37932 *isomorphism types of simple* 7-(26, 9, λ) *designs with automorphism group* $P\Gamma L(2,25)$, *in each case, for* $\lambda = 54$ *and* 63 *respectively. There exist many isomorphism types of simple* 7-(26, 9, 81) *designs with automorphism group* $P\Gamma L(2,25)$. *There exist simple* 7-(27, 9, 60) *designs. There exist exactly* 1 *isomorphism type of simple* 7-(24, 8, 4) *designs and exactly* 138 *isomorphism types of simple* 7-(24, 8, 5) *designs with automorphism group* $PSL(2,23)$. *There exist at least* 590, 126, *and* 63 *isomorphism types of simple* 7-(24, 8, λ) *designs for* $\lambda = 6$, 7, *and* 8 *respectively, with automorphism group* $PSL(2,23)$. *In addition there exist exactly* 4 *isomorphism types of simple* 7-(24, 8, 8) *designs with automorphism group* $PGL(2,23)$. *There exist exactly* 113 *isomorphism types of simple* 7-(24, 9, 40) *designs, there exist exactly* 5463 *isomorphism types of simple* 7-(24, 9, 48) *designs, and there exist at least* 15335 *isomorphism types of simple* 7-(24, 9, 64) *designs with automorphism group* $PGL(2,23)$. *There exist simple* 7-(25, 9, λ) *designs for* $\lambda = 45$, 54, 72.

The full set of solutions for 7-(24, 8, λ) designs with prescribed group of automorphisms $PSL(2,23)$ is not yet known, but this set seems to be very large.

The 7-(27, 9, 60) and 7-(25, 9, λ) designs are constructed by a method of Tran van Trung [26] from other designs, see also D. L. Kreher [14]. However, we do not get results on the number of isomorphism types in these cases. In general, we also do not know the automorphism group of these designs. This means that we could not directly construct these designs with the Kramer-Mesner method. Using standard constructions like forming the derived or the residual designs we obtain a lot of new parameter sets for simple t-designs with $t < 7$ from Theorem 1.1. Remarkably, there result a lot of parameter sets of 5-designs with an odd number of points.

While we were constructing new designs, we also proved a lot of non-existence results which are not included in this overview.

We further mention that the 7-(26, 8, 6) designs with automorphism group $PGL(2,25)$ are the only ones admitting $PSL(2,25)$ as an automorphism group. Besides the 7-designs we also found some new 6-designs.

Theorem 1.2. *There exist exactly* 9, 49, 476, 1284, *and* 3069 *isomorphism types of simple* 6-$(24, 8, \lambda)$ *designs with automorphism group* $PGL(2, 23)$ *for* $\lambda = 36, 45, 54, 63,$ *and* 72, *respectively. There exist exactly* 242 *isomorphism types of simple* 6-$(25, 8, 36)$ *designs, exactly* 10008 *isomorphism types of simple* 6-$(25, 8, 45)$ *designs, and there exist simple* 6-$(25, 8, \lambda)$ *designs for* $\lambda = 54, 63, 72, 81$, *admitting the automorphism group* $PGL(2, 23)p$ *in each case.* $PGL(2, 23)p$ *is the permutation group on* 25 *points which is obtained from* $PGL(2, 23)$ *in its natural action on* 24 *points by adding an additional fixed point.*

There are no 6-$(25, 8, \lambda)$ designs admitting $PGL(2, 23)p$ for $\lambda = 9, 18, 27$.

2. THE KRAMER-MESNER METHOD REVISITED

A standard tool for constructing t-designs goes back to Kramer and Mesner [13]. In this method, one assumes a group A of automorphisms of the desired t-(v, k, λ) designs. In other words, this particular group A is prescribed and one is looking for designs admitting that group as a symmetry group. Of course, this is a risky business as the set of designs satisfying this additional condition may be empty. However, the assumption of such a group of automorphisms reduces the size of the problem enormously, allowing to tackle problems which would otherwise be too hard to attack.

Let us sketch this method briefly. The group A is a permutation group on the underlying set V which we take as point set for our design. Moreover, we also have A acting on k-subsets of V. A design $(V, \mathcal{B})$ admits A as an automorphism group if and only if A maps blocks of the design onto blocks, that is, the set of blocks of the design consists of full k-orbits of A:

$$\mathcal{B} = K_1^A \,\dot\cup\, K_2^A \,\dot\cup \ldots \dot\cup\, K_r^A$$

with $K_1, \ldots, K_r$ being base blocks of the block orbits in $\mathcal{B}$, respectively.

Now one has to consider the covering of t-subsets of V by blocks of the putative design. For T any t-subset and K any k-subset, let T be contained in exactly $m(T, K^A)$ k-subsets of K^A. It is easy to see that $m(T, K^A) = m(T', K^A)$ for $T' = T^\alpha$ with an arbitrary $\alpha \in A$, that is, the number $m(T, K^A)$ is independent of the choice of the set T in its A-orbit.

To ensure the conditions of a design it is sufficient to check that a collection of k-orbits covers a set of representatives $T_1, \ldots, T_h$ of t-orbits exactly λ times each. In order to ensure this condition one forms an $h \times r$ matrix $M_{t,k}^A$ indexed by t-orbit and k-orbit representatives, respectively, with $m(T_i, K_j^A)$ in its i, j-th position. We call this matrix Kramer-Mesner matrix.

Choosing a collection of k-orbits can be interpreted as multiplying the matrix by a 0/1-vector x of length r, where a 1 means that the corresponding k-orbit should belong to the design. Such a collection of k-orbits forms a designs if and only if $M_{t,k}^A x = (\lambda, \ldots, \lambda)^t$ where λ is repeated h times on the right hand side. Non-simple designs are obtained by allowing solution vectors with integer entries larger than 1.

At Bayreuth, the authors are developing a software package DISCRETA for the construction and handling of discrete structures, with t-designs being an outstanding but not exclusive topic of research. Using a double coset construction technique, the Kramer-Mesner matrices are evaluated using a new implementation of the Leiterspiel (snakes and ladders) [21]. Moreover, the system provides an LLL based solver of systems of Diophantine linear equations with unknowns only in 0/1 (cf. also [16, 27]). See [3] for a short overview on the algebraic background and the general principles which are applied. A major improvement of the solver is to start with the computation of an LLL-reduced integer basis of the kernel of the given Kramer-Mesner system and then to enumerate all integer linear combinations of these basis vectors yielding 0/1-solutions of the Kramer-Mesner matrix. We apply improved algorithms for LLL-reductions (cf. [23, 24]) and base the explicit enumeration of solutions on an algorithm in [11]. Last but not least we may leave λ open as it is considered as a variable in the system of equations. Thus, the system also suggests appropriate parameters which sometimes lead to unexpected results.

A decisive feature of DISCRETA is a graphical user interface written in OSF-MOTIF. All actions can be controlled from menus via mouse clicking. Moreover, the system has a variety of groups available, most of them being parameterized by integer numbers. For example, we can form different kinds of linear groups for reasonable dimension and size of the finite field. DISCRETA allows to build up new groups from these using standard constructions like forming the direct sum or the direct product. Moreover, one may choose between different equation solvers, e.g. the LLL algorithm, a solver written by B. D. McKay, and a linear programming package lpsolve [1]. The computed data can be stored and be reported in various formats like TEX or HTML. A database of design parameter sets is also included.

Let us now point out some additional remarks which might help simplifying the construction problem. First, we may enlarge the Kramer-Mesner matrix by one further row, containing the orbit lengths of the k-orbits. We know in advance that a t-(v, k, λ) design has exactly

$$b = \frac{\binom{v}{t}}{\binom{k}{t}} \cdot \lambda$$

blocks. So, this additional row in the system ensures that the orbit lengths in the design sum up to b. Often one can conclude that not all k-orbits in the design can have full length $|A|$. So, one may start with choosing among the short orbits, that is those with length less than $|A|$. Suppose we take $P\Gamma L(2, 32)$ as an automorphism group of a design with paramters 7-(33,8,10), cf. [3]. The number of blocks in such a design is $b = 5340060$. The orbits on 8-subsets have lengths 163680, 81840, and 20460. Let a_i be the number of orbits of length $|A|/i$ in the design. Dividing

$$b = a_1 \cdot 163680 + a_2 \cdot 81840 + a_8 \cdot 20460$$

by 20460 we get

$$261 = a_1 \cdot 8 + a_2 \cdot 4 + a_8.$$

Obviously, $a_8 \neq 0$ and since there exists only one orbit of this length, $a_8 = 1$. So, there remains the restriction

$$65 = a_1 \cdot 2 + a_2.$$

In the solution presented in [3] we have $a_1 = 27$ and $a_2 = 11$.

In the case that $A = PSL(2, p)$ is prescribed as an automorphism group of a Steiner system with parameters 5-$(p + 1, 6, 1)$, Grannell, Griggs, and Mathon have shown in [9] that if 5 is not a divisor of $|A|$ then each 5-set has a trivial stabilizer in A. In this case, any 6-set may have a stabilizer of order at most 6.

Consequently, there are only orbits of lengths $|A|/n$ for $n = 1, 2, 3, 6$ on the set of 6-subsets when $p \in \{11, 23, 47, 71, 83, 107, 131\}$. So we obtain the equation

$$(p+1)p(p-1)(p-2)(p-3)/6! = b = (p+1)p(p-1)/2 \cdot (a_1 + a_2/2 + a_3/3 + a_6/6)$$

where the design has a_i 6-orbits of length $|A|/i$ for $i = 1, 2, 3, 6$. This equation reduces to

$$(p-2)(p-3)/60 = 6 \cdot a_1 + 3 \cdot a_2 + 2 \cdot a_3 + a_6.$$

If $p \not\equiv 3 \bmod 8$ then 6 does not divide the left hand side so that some a_i for $i > 1$ must be greater than 0. Further restrictions may be deduced in special cases. For example, $p = 47$ yields $a_6 \equiv a_3 \bmod 3$.

B. D. McKay remarked that the additional equation involving the lengths of orbits can be interpreted as an approximation of all 0-sets by the k-sets of the desired design. This parallels the construction of Kramer and Mesner to describe the possibilities of approximating t-subsets and leads to an interesting generalization: For $0 < s < t$, one can look at the approximation of s-sets by k-orbits. These additional equations appear naturally when setting up the Kramer-Mesner systems for the designs with reduced t. It is well known that a t-design is also a s-design for $0 \le s < t$. The resulting enlarged system of linear equations now has different values of λ on the right hand side but must have the same 0/1-solution vectors.

3. ISOMORPHISM PROBLEMS

The second important remark concerns the question of isomorphism of designs. Often, a more or less complicated system of invariants is used to classification purposes. Knowledge about the full automorphism groups is considered as a poor means of classification (cf. [7]). However, in [21] the full automorphism groups are used as a tool for determining the isomorphism types. It is easy to see that two designs defined on the same point set with the same automorphism group can only be mapped upon each other by an element of the normalizer of that group. Unfortunately, the Kramer-Mesner method only finds designs having *at least* the prescribed automorphism group A. So, in [21] a Moebius inversion technique is applied to find the designs with given full automorphism group and then the above argument is applied. However, this requires a thorough knowledge of the full lattice of subgroups between A and S_V. So, in Theorem 1.1 we claimed the existence of 7 isomorphism types of 7-(26, 8, 6) designs. In fact, we found twice as many solutions of the system of equations. In addition, we found that there were no solutions for the group $P\Gamma L(2, 25)$ for this parameter set. Since this is the only proper subgroup of S_{26} containing $PGL(2, 25)$ (see, for example [2]), all our solutions have the latter group as their full automorphism group. The normalizer of $PGL(2, 25)$ is $P\Gamma L(2, 25)$, which has orbits of length $2 = |P\Gamma L(2, 25)/PGL(2, 5)|$ on the set of designs with automorphism group $PGL(2, 25)$. We remark that there are no additional solutions for the group $PSL(2, 25)$, such that no overgroup of $PSL(2, 25)$ different from $PGL(2, 25)$ appears as the full automorphism group of a design with these parameters. The most simple case is when $P\Gamma L(2, 25)$ is known to be an automorphism group. Then by this argument all solutions of the system of equations are pairwise non-isomorphic designs. This applies to the 3989 solutions for 7-(26, 9, 54). Also, the cases where $PGL(2,23)$ is a prescribed automorphism group can be handled in this way. Interestingly, in some important situations this approach can be much simplified, so that we can solve isomorphism problems of designs with only very local knowledge of subgroups.

Theorem 3.1. *Let G be a finite group acting on a set X. Let $x_1, x_2 \in X$ and $g \in G$ such that $x_1^g = x_2$. Let a Sylow subgroup P of G be contained in the stabilizers $N_G(x_1)$ and $N_G(x_2)$. Then $x_1^n = x_2$ for some $n \in N_G(P)$.*

This result is an slight generalization of Hilfssatz IV 2.5 in [10]. For convenience, we repeat the proof here.

Proof. Since $P^g \leq N_G(x_1)^g = N_G(x_1^g) = N_G(x_2)$ and also $P \leq N_G(x_2)$, there is some $h \in N_G(x_2)$ such that $P^g = P^h$ by the Sylow Theorem. Then $gh^{-1} = n \in N_G(P)$ and $g = nh$. Therefore $x_2 = x_1^g = x_1^{nh}$ and $x_1^n = x_2^{h^{-1}} = x_2$. □

Let us apply this theoreom to the case of t-designs. Here, G is the full symmetric group S_V inducing an action on the set X of all t-designs with point set V. Assume the prescribed automorphism group A contains a Sylow subgroup P of S_V. Then by Theorem 3.1 two designs x_1, x_2 having A as an automorphism group may be mapped upon each other by a permutation g only if there exists some $n \in N_{S_V}(P)$ which maps x_1 onto x_2. If even $N_{S_V}(P)$ is contained in A then all designs fixed by A are pairwise not isomorphic. So, in this case the solutions of the system of linear equations given by the Kramer-Mesner matrix form a full set of representatives from all isomorphism types of designs admitting A as full automorphism group.

If, for example, A is the holomorph of C_{19}, that is, the normalizer of C_{19} in S_{19} which is isomorphic to the semidirect product of C_{19} with its automorphism group with respect to the natural action, then all designs on 19 points admitting A as an automorphism group are pairwise non-isomorphic. Thus, there are exactly 255 isomorphism types of 5-(19, 6, 4)-designs and 17193 isomorphism types of 5-(19, 6, 6)-designs admitting this automorphism group.

An important case where the condition of Theorem 3.1 is fulfilled is the projective group $PGL(2,p)$ for some prime p. This group is the permutation representation of the general linear group $GL(2,p)$ on the set of all $p+1$ subspaces of dimension 1 of the underlying vector space $V = V(2,p)$. It has order $(p+1)p(p-1)$ and contains a Sylow p-subgroup of the full symmetric group S_{p+1}. The normalizer N of a 1-dimensional subspace T of V in $GL(2,p)$ has order $p(p-1)^2$ and contains the centralizer of T and V/T as a normal subgroup. This centralizer is just of order p and therefore a normal subgroup of N. If we reduce modulo the center Z of $GL(2,p)$ which is of order $(p-1)$ we obtain that PZ/Z is a normal subgroup of NZ/Z and NZ/Z has order $p(p-1)$. Now this is just the order of the normalizer of a Sylow p-subgroup of S_{p+1} such that $PGL(2,p)$ contains the normalizer of a Sylow subgroup of S_{p+1}. This means that whenever we construct objects with $PGL(2,p)$ acting as a group of automorphisms we know that all these objects are pairwise nonisomorphic.

Let us consider the famous Witt 5-(24, 8, 1) design, of which the automorphism group M_{24} contains $PSL(2,23)$ as a subgroup. This subgroup acts as a group of automorphisms on that design. Since $PGL(2,23)$ is not contained in M_{24}, there must be a second design fixed by $PSL(2,23)$ and interchanged with the first by $PGL(2,23)$. The union of both designs is a 5-(24, 8, 2) design with automorphism group $PGL(2,23)$. This design consists of just one orbit of $PGL(2,23)$ on the set of 8-subsets. The same situation occurs with M_{12} and $PGL(2,11)$ as the corresponding groups. These designs must have been contained in those reported by [12] to exist. We only note that the systematic construction here results in *block-transitive* designs.

We note that there are exactly two solutions for a 5-(24, 7, 3) design with $PSL(2,23)$ as a group of automorphisms, so that there is just one isomorphism type of this kind. These solutions are block-transitive, which is no longer true for designs with larger values of λ. Exact numbers of isomorphism types for $(t,k) \in \{(3,4), (3,5), (4,5), (3,6), (4,6), (5,6)\}$, and also for larger values of λ, are contained in [21].

In a series of papers [8], [9], [6], Grannell, Griggs and Mathon have shown that in many cases only $PSL(2,p)$ appears as a group of automorphisms of Steiner systems. As we have shown, $PGL(2,p)$ contains the required normalizer of a Sylow subgroup of S_{p+1}. Therefore only the action of

$$PGL(2,p)/PSL(2,p) \cong C_2$$

on the set of solutions of the Kramer-Mesner system has to be taken into account. This explains why in [8], [9], [6], and [21], a representative from the non-trivial coset of $PSL(2,p)$ in $PGL(2,p)$ already suffices to distinguish between the isomorphism types of Steiner systems constructed with automorphism group $PSL(2,p)$.

All our claims about the number of isomorphism types of designs admitting as automorphism group $PSL(2,23)$ rely on the above argument and a complete construction of all solutions of the corresponding Kramer-Mesner system of Diophantine equations by our program. The situation is more difficult in the case of $PGL(2,23)p$. This group still contains a 23-Sylow subgroup P of S_{25}, and the normalizer N of P maps the fixed points of P onto fixed points. Thus, N is the direct product of S_2 and $Hol(C_{23})$, the latter being contained in $PGL(2,23)$. So, if two designs of this type are isomorphic, the transposition τ of the two fixed points of P must interchange them. In this case, τ must also interchange their automorphism groups, A and B say. Since $PGL(2,23)p$ is contained in A, the conjugation by τ moves $PGL(2,23)p$ onto a subgroup of B. Also $PGL(2,23)p$ is contained in B. But $PGL(2,23)p)$ and its conjugate together generate the full group S_{25}. Since only the trivial and the complete design have this group as an automorphism group we deduce that all designs fixed by $PGL(2,23)p$ are pairwise non-isomorphic.

4. DESCRIPTION OF DESIGNS

The following tables display designs for some of the parameter sets listed in Theorem 1.1 and Theorem 1.2. For a complete treatment, the reader should consult our electronic publication on the web (see below). In the tables, the first column shows orbit representatives of all k-orbits. The length of the orbits is indicated in the second column. For each λ, there are some columns of 0/1-entries. Each column stands for one design. Each of these columns is a solution vector of the Kramer-Mesner system of equations. Thus, an entry 1 in the i-th row means that the i-th orbit belongs to the design described by this solution vector. A 0-entry means that this orbit does not belong to that design.

For small numbers of solutions we completely list all isomorphism types. To indicate completeness of the solutions, we mark that column by a "$\diamond$"-sign. In the other cases only 5 solutions are given to enable the analysis of such designs. A more complete listing can be found in the electronic tables of the journal or on the web pages of the authors. The addresses are:

`http://www.emba.uvm.edu/~jcd/reports/282/pub_7designs_jcd.html`
`http://www.mathe2.uni-bayreuth.de/betten/PUB/pub_7designs_jcd.html`

4.1 Representation of the automorphism groups

We use the following permutation representation of $PGL(2,23)$, a group of order 12144. Generators are the permutations

$$\begin{aligned}\alpha &= (3\ 7\ 4\ 12\ 6\ 22\ 10\ 19\ 18\ 13\ 11\ 24\ 20\ 23\ 15\ 21\ 5\ 17\ 8\ 9\ 14\ 16)\\ \beta &= (3\ 16\ 14\ 9\ 8\ 17\ 5\ 21\ 15\ 23\ 20\ 24\ 11\ 13\ 18\ 19\ 10\ 22\ 6\ 12\ 4\ 7)\\ \gamma &= (2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 22\ 23\ 24)\\ \delta &= (1\ 3\ 14\ 10\ 8\ 16\ 6\ 12\ 5\ 20\ 9\ 23\ 4\ 18\ 7\ 22\ 15\ 21\ 11\ 19\ 17\ 13\ 24)\end{aligned}$$

The permutations β^2, γ, and δ generate $PSL(2,23)$ of order 6072. The group $PGL(2,23)p$ results from adding the fixed point 25 to each of the generators.

We use the following permutation representation of $P\Gamma L(2,25)$, a group of order 31200. Generators are the permutations

$$\begin{aligned}\alpha &= (1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 22\ 23\ 24)\\ \beta &= (1\ 17\ 14\ 15\ 10)(2\ 5\ 13\ 22\ 3)(4\ 11\ 9\ 19\ 8)(6\ 18\ 12\ 25\ 24)(7\ 21\ 23\ 16\ 20)\\ \gamma &= (1\ 8\ 4\ 17\ 3)(2\ 21\ 22\ 19\ 11)(5\ 16\ 20\ 13\ 15)(6\ 12\ 26\ 24\ 18)(7\ 10\ 9\ 14\ 23)\\ \delta &= (1\ 5)(2\ 10)(3\ 15)(4\ 20)(7\ 11)(8\ 16)(9\ 21)(13\ 17)(14\ 22)(19\ 23)\end{aligned}$$

The permutations α, β, and γ generate the group $PGL(2,25)$ of order 15600.

4.2 Designs with automorphism group $PGL(2,23)$

TABLE I. 7-(24,9,λ) Designs

orbits on 9-subsets of V		solutions		
representative	length	$\lambda = 40$	$\lambda = 48$	$\lambda = 68$
1 2 3 4 5 6 7 11 12	6072	11111	11111	11111
1 2 3 4 5 6 7 8 9	6072	00100	10000	11111
1 2 3 4 5 6 7 9 12	12144	00110	00001	11001
1 2 3 4 5 6 7 12 18	12144	01001	00000	00000
1 2 3 4 5 6 7 10 20	6072	00000	00000	00000
1 2 3 4 5 6 7 8 13	12144	10000	00010	00000
1 2 3 4 5 6 7 8 12	12144	00000	10000	10110
1 2 3 4 5 6 7 12 14	12144	11011	00111	11111
1 2 3 4 5 6 7 10 12	6072	10110	00000	00000
1 2 3 4 5 6 7 8 10	12144	00001	00111	01101
1 2 3 4 5 6 7 8 11	12144	00000	11000	00010
1 2 3 4 5 6 7 8 14	12144	00000	01100	00000
1 2 3 4 5 6 7 8 15	12144	10010	01011	00000
1 2 3 4 5 6 7 8 16	12144	01100	00000	01011
1 2 3 4 5 6 7 11 13	12144	00100	01100	00010
1 2 3 4 5 6 7 13 17	12144	00001	01000	10110
1 2 3 4 5 6 7 13 19	6072	01000	11100	00000
1 2 3 4 5 6 7 13 16	12144	01011	00000	11111
1 2 3 4 5 6 7 10 11	12144	01000	10100	01101
1 2 3 4 5 6 7 13 18	12144	00000	11001	10000
1 2 3 4 5 6 7 10 13	12144	00110	01011	01100
1 2 3 4 5 6 7 13 14	12144	00000	10010	11111
1 2 3 4 5 6 7 10 19	12144	00111	00100	00001
1 2 3 4 5 6 7 13 15	6072	00000	11100	11111
1 2 3 4 5 7 9 13 14	12144	00000	00001	11111
1 2 3 4 5 7 8 10 13	12144	11100	11000	10110
1 2 3 4 5 7 8 9 22	12144	10001	00000	11010
1 2 3 4 5 7 9 10 22	12144	11100	10110	01111
1 2 3 4 5 7 9 13 21	12144	00001	00000	00000
1 2 3 4 5 7 9 13 18	12144	01000	00010	11000
1 2 3 4 5 7 8 13 20	12144	00010	00000	00001
1 2 3 4 5 7 8 9 13	12144	00000	00101	00001
1 2 3 4 5 6 8 10 14	6072	00000	00011	11111
1 2 3 4 5 6 9 10 21	6072	00001	11111	11111
1 2 3 4 5 7 8 10 11	12144	00110	01000	01000
1 2 3 4 5 7 8 13 21	4048	11111	00000	11111
1 2 3 4 5 7 8 13 17	6072	11111	11111	11111
1 2 3 4 5 6 9 13 17	6072	00000	11111	11111
1 2 3 4 5 7 8 14 16	12144	10010	00110	01100
1 2 3 4 5 6 7 14 16	12144	00100	10000	00000
1 2 3 4 5 7 10 11 14	6072	00000	11111	00000
1 2 3 4 5 7 10 13 17	6072	00001	00000	00000
1 2 3 4 5 6 7 9 10	12144	00000	00000	10001
1 2 3 4 5 6 7 9 11	12144	11010	00011	11100
1 2 3 4 5 6 7 9 14	12144	00001	11110	00000
1 2 3 4 5 6 7 9 17	12144	00000	11100	00110
1 2 3 4 5 6 7 9 18	6072	11000	00000	11111
1 2 3 4 5 6 7 9 23	2024	11111	00000	11111
1 2 3 4 5 6 7 10 14	12144	00110	00000	11100
1 2 3 4 5 6 7 10 15	12144	10101	01011	00000
1 2 3 4 5 6 7 10 16	12144	00000	01101	11111
1 2 3 4 5 6 7 10 17	12144	00001	01110	10010
1 2 3 4 5 6 7 10 18	12144	01010	10000	01011
1 2 3 4 5 6 7 10 22	6072	11011	10000	11111
1 2 3 4 5 6 7 11 17	12144	10001	00011	00011
1 2 3 4 5 6 7 14 15	12144	01000	11101	00100
1 2 3 4 5 6 7 14 17	12144	00100	00100	11101
1 2 3 4 5 6 7 14 18	2024	11111	00000	11111
1 2 3 4 5 6 7 15 16	12144	10000	00001	11010
1 2 3 4 5 6 7 15 17	6072	01000	00011	11111
1 2 3 4 5 7 8 18 21	12144	11001	10101	01011
1 2 3 4 5 7 8 14 22	12144	00100	01000	00100
1 2 3 4 5 6 10 12 13	12144	10000	00000	00010
1 2 3 4 5 7 11 13 18	2024	11111	00000	11111
1 2 3 4 5 7 10 11 13	6072	01010	10011	11111
1 2 3 4 5 6 8 12 14	12144	00000	10010	00000
1 2 3 4 5 6 8 9 23	12144	10010	00001	10101
1 2 3 4 5 7 8 9 10	6072	00110	11111	00000
1 2 3 4 5 6 10 13 17	12144	00000	00011	00001
1 2 3 4 5 6 8 14 21	12144	00111	01100	11011
1 2 3 4 5 7 8 10 22	12144	01000	10110	11100
1 2 3 4 5 6 9 10 17	12144	10000	00000	10100
1 2 3 4 5 7 8 16 22	6072	11111	11111	11111
1 2 3 4 5 6 8 9 10	12144	01010	01000	11011
1 2 3 4 5 6 8 9 12	12144	10010	10000	00110
1 2 3 4 5 6 8 9 13	12144	01000	10100	01110
1 2 3 4 5 6 8 9 15	12144	01100	10000	00001
1 2 3 4 5 6 8 9 16	6072	01101	01111	11111
1 2 3 4 5 6 8 9 17	12144	00000	01011	11110
1 2 3 4 5 6 8 9 18	12144	10111	00100	00110
1 2 3 4 5 6 8 9 22	12144	01110	00001	00100
1 2 3 4 5 6 8 10 13	4048	11111	00000	11111
1 2 3 4 5 6 8 10 16	12144	10001	10000	01000
1 2 3 4 5 6 8 10 17	12144	00110	11100	01100
1 2 3 4 5 6 8 10 21	12144	00110	10000	10001
1 2 3 4 5 6 8 10 22	12144	00100	01010	10010
1 2 3 4 5 6 8 12 15	6072	01011	01100	11111
1 2 3 4 5 6 8 12 17	12144	00001	00100	11000
1 2 3 4 5 6 8 12 18	12144	00000	00100	11111
1 2 3 4 5 6 8 12 21	12144	10000	01111	11001
1 2 3 4 5 6 8 12 22	12144	00000	01110	01111
1 2 3 4 5 6 8 16 22	6072	00000	11111	00000
1 2 3 4 5 6 8 17 18	12144	11001	10100	00000
1 2 3 4 5 6 8 17 21	12144	00000	00010	00001
1 2 3 4 5 6 8 17 22	12144	00000	00010	10010
1 2 3 4 5 6 8 21 22	12144	00001	00000	01001
1 2 3 4 5 6 9 10 12	6072	11110	11111	11111
1 2 3 4 5 6 9 10 13	12144	01101	10010	00111
1 2 3 4 5 6 9 10 14	12144	11100	00001	10111
1 2 3 4 5 6 9 10 18	12144	10000	10001	00000
1 2 3 4 5 6 9 10 19	12144	01000	00001	10001
1 2 3 4 5 6 9 13 14	12144	11101	10000	00111
1 2 3 4 5 6 9 13 18	12144	00010	00000	01000
1 2 3 4 5 6 9 13 19	12144	00000	00000	00001
1 2 3 4 5 6 9 13 21	12144	00000	11001	00000
1 2 3 4 5 6 9 14 19	12144	01010	00010	11111
1 2 3 4 5 6 9 14 21	12144	00000	00001	01010
1 2 3 4 5 6 10 12 14	12144	00001	00000	11001
1 2 3 4 5 6 10 13 14	12144	00110	00000	00010
1 2 3 4 5 6 10 13 18	12144	10000	01100	00010
1 2 3 4 5 6 10 13 19	12144	11001	11001	01000
1 2 3 4 5 6 10 13 21	6072	10000	11111	11111
1 2 3 4 5 6 10 14 17	12144	01000	10000	10100
1 2 3 4 5 6 10 17 19	12144	00100	00011	10100
1 2 3 4 5 8 9 10 13	6072	10111	11111	11111
1 2 3 4 5 7 8 11 16	12144	00010	01110	11101
1 2 3 4 5 7 9 14 21	12144	00000	01110	10100
1 2 3 4 5 7 8 9 16	12144	00000	10001	00001
1 2 3 4 5 7 8 9 14	12144	10000	01101	11110
1 2 3 4 5 7 9 16 22	12144	00001	00010	00111
1 2 3 4 5 7 9 16 17	6072	00110	00000	00000
1 2 3 4 5 7 8 9 18	12144	00001	01011	10011
1 2 3 4 5 7 8 9 17	12144	00000	00000	00000
1 2 3 4 5 7 8 10 18	6072	00111	11111	11111
1 2 3 4 5 7 8 14 21	6072	00011	10000	11111

TABLE II. 6-(24,8,λ) Designs

orbits on 6-subsets of V		solutions				
representative	length	λ = 36 *	λ = 45 *	λ = 54	λ = 63	λ = 72
1 2 3 4 5 6 7 12	6072	010000000	111000001100111000000100000010100000000110010011 0	00000	00000	11111
1 2 3 4 5 6 7 8	6072	001000101	001000011100110010000000001000011001000000101100 0	11111	11111	00000
1 2 3 4 5 6 7 13	12144	000010010	000010000000000010101101100000000011011000000000 1	10001	00011	00010
1 2 3 4 5 7 9 13	12144	100000100	101100100000000010000000011100000100111000011000 0	01010	01001	10101
1 2 3 4 5 7 8 13	12144	000000000	000110100000000001010000000110111000000010100000 0	00000	00001	10011
1 2 3 4 5 8 10 20	3036	111010111	101000010000101101001101110111100110110101010011 0	01110	11100	01010
1 2 3 4 5 7 15 19	12144	000101000	010001010011001000000010110000000101000000100011 01	00010	11110	01000
1 2 3 4 5 7 15 17	12144	000000000	000001001111000100100000001001000010100100001000 0	10101	10100	01100
1 2 3 4 5 6 7 9	6072	010110001	000101100111001100100000111110101010000000000011 0	00001	00010	11111
1 2 3 4 5 6 7 10	12144	000000000	010001000010010000010000000000100000000000000100 10	01110	11110	00100
1 2 3 4 5 6 7 11	6072	101101000	000011111001100001111111010010111100100111101100 1	11100	11000	00011
1 2 3 4 5 6 7 14	12144	000000000	000000000000000000000000000010000001001001000100 1	01000	00110	00000
1 2 3 4 5 6 7 15	12144	001001110	100000000000000010000000001101000000011000010000 0	00001	00000	11111
1 2 3 4 5 6 7 16	6072	100010000	000100000111000000101100000000000100001010100001 0	00100	00001	11100
1 2 3 4 5 7 11 13	12144	000011001	010000000000001000001010010000000000000000001000 000	10010	00100	01000
1 2 3 4 5 7 9 10	12144	010000001	111100000000111100100110111011101111011001000100 1	00101	10110	00101
1 2 3 4 5 7 13 16	6072	000100110	111010010111010111001001000000000000000000100010 10	11001	00010	11100
1 2 3 4 5 7 10 11	12144	001100010	000000000000000001000100100000000000000000000001 10	00100	11011	10100
1 2 3 4 5 7 10 13	12144	110000001	000000001110011000000000000000111000100110011011 0	11000	01000	11011
1 2 3 4 5 6 8 14	6072	011100000	000000000000000111111111100000000000000111111111 11	00000	11111	00000
1 2 3 4 5 7 8 22	6072	100011000	000011111111001000000111100000000011110000100000 000	00011	00001	11111
1 2 3 4 5 6 13 17	3036	011111111	111111111111111111111111100000000000000000000000 0	11111	00000	11111
1 2 3 4 5 6 8 9	12144	010001000	000100000000000000000000000000001001001000000100 00	00000	01101	11001
1 2 3 4 5 6 8 10	12144	000000010	011000000000000000000000000100000101000001000000 0	10010	11000	10110
1 2 3 4 5 6 8 12	12144	000000001	000001000010000000000000000000000000000010000000 0	00010	10100	11011
1 2 3 4 5 6 8 15	3036	011111111	111111111111111111111111110000000000000000000000 0	11111	00000	11111
1 2 3 4 5 6 8 16	6072	100100000	000000111100100001011001110000100110110000001001	11011	00001	00100
1 2 3 4 5 6 8 17	12144	100000100	000011000000010000000101000111000001000100110010	00000	11110	00011
1 2 3 4 5 6 8 18	6072	100000000	110010100000000100100000001111111100000101000100	00100	00000	00011
1 2 3 4 5 6 8 21	12144	000010000	000000000010010100000000000000000001000001000000	11100	00010	00100
1 2 3 4 5 6 8 22	12144	000000000	111010000000000100000000000100010000001010000001	01100	00000	01000
1 2 3 4 5 6 8 23	3036	000000000	111111111111111111111111111111111111111111111111	00000	11111	00000
1 2 3 4 5 6 9 10	12144	100100110	000100001000001101001010100101001110101100001100	10110	10100	11101
1 2 3 4 5 6 9 13	12144	000010100	000000000110001010000001100000010110100000000100	00011	00101	11010
1 2 3 4 5 6 9 14	12144	000000001	100000011000110000010100011101000100011011010110	11000	10000	01101
1 2 3 4 5 6 9 19	12144	001000000	101010110001000011100110100000000001011001000010	00001	00011	00010
1 2 3 4 5 6 9 22	3036	000000000	111111111111111111111111111111111111111111111111	00000	11111	00000
1 2 3 4 5 6 10 12	6072	011100010	110100010000111111111111000111010010000010010101	00000	11111	11000
1 2 3 4 5 6 10 13	12144	000100100	100110110011000011010000000101000000000001100000	00000	00100	00000
1 2 3 4 5 6 10 14	12144	100000000	000000001100010000001000100001010001000000100010	10001	00001	11101
1 2 3 4 5 6 10 17	12144	100000000	000001110111000000000000001100000001100000100000	00010	01001	00000
1 2 3 4 5 6 10 18	12144	000001001	011100100000101100000001011100011100010000100000	01110	11001	11101
1 2 3 4 5 6 10 19	6072	110001011	111110010000000100000010101101100101001100000101	00101	11000	10101
1 2 3 4 5 6 10 21	6072	011101001	000000111111100001111110000000001100000011111 1	11111	11111	00000
1 2 3 4 5 6 13 14	6072	100011100	001110010011000010110101000111011100011011011 0	00010	11111	01010
1 2 3 4 5 6 13 18	6072	000010000	001001011100011011000100111000000011111000000100	11111	00001	11111
1 2 3 4 5 8 10 13	12144	000000000	000001001000101100000000000010000000000001001000	01100	10110	01001
1 2 3 4 5 7 9 16	12144	001000000	000010110100000001100000100100011100000001010000	00110	01001	00011
1 2 3 4 5 9 10 12	6072	000001000	111110101011011000010110100000110010101010100001	01001	10011	01100
1 2 3 4 5 8 9 10	12144	000001000	110100010000000001001100000100011010010110010100	01000	00010	01000
1 2 3 4 5 7 8 18	12144	010100100	101000110111001101111001010101000010000111000000 10	11100	11001	00110
1 2 3 4 5 7 8 20	6072	011110100	010101101000001011010001010100110100000001110100 1	00010	01110	10000
1 2 3 4 5 7 10 17	12144	000010000	001011000001100001000000000000000000000000000100 10	00000	00000	10011
1 2 3 4 5 7 9 22	12144	001000000	000000000000000000000000000101000000000000000000	00011	00101	11011
1 2 3 4 5 7 14 20	6072	001000000	000110000011010011001000000000011100001110000011 1	00000	11111	00000
1 2 3 4 5 7 18 20	6072	100100100	001011000100100000101000000011001000101100010010	11111	00000	11111
1 2 3 4 5 7 8 10	12144	000001001	000001000110010000111100010000000000111000000100 01	01001	10101	11100
1 2 3 4 5 7 8 21	12144	100000000	010000000000000100101000000000000100001000000010 10	11000	01000	01001
1 2 3 4 5 7 10 22	12144	000111110	000000000000000000000001110000000010000001001000	10000	00000	00111
1 2 3 4 5 7 8 9	12144	000000000	000000001000101100000001000001000000000100100000 0	11100	10100	01101
1 2 3 4 5 7 8 14	12144	000110010	000100000100010000000001011011100111000000001100	00110	00010	11110
1 2 3 4 5 7 9 17	12144	010000000	001110110100010000000111000010000000000010001000	10000	01011	10100
1 2 3 4 5 7 8 17	6072	110100001	100000000011111010101000000000100100111100001001 1	01011	11110	10001
1 2 3 4 5 7 9 14	6072	100101111	010011000001011111010111000100110001011101000100 1	00101	01011	11011
1 2 3 4 5 7 10 14	6072	011010010	110100110000000001001011110100100111110101000000	10011	01010	00110
1 2 3 4 5 7 10 16	12144	010000000	101010101000100100101100000111001100000001001100 0	00001	00100	10011
1 2 3 4 5 7 9 18	12144	100000010	000000001000100010001000000000001000000010001001 00	10000	10000	10000
1 2 3 4 5 7 11 17	12144	001000000	001000000001001101000011000010001001000000100011	00101	10001	00111
1 2 3 4 5 7 8 11	6072	000000111	010111000100110010010101001000000000011111100101	10000	00110	01000
1 2 3 4 5 7 9 21	12144	011010000	110001000110000010000000100101011101100010000111 0	00001	01010	10110
1 2 3 4 5 7 8 16	12144	001000010	001000101001000101000000010100001000010001000000 1	00110	01001	00010
1 2 3 4 5 7 11 19	12144	010000101	000100010100011100110000000001000000001000000000	11000	11110	00011
1 2 3 4 5 7 11 18	6072	000010010	000100011000011010100100100010001001110001000000	00001	00011	11111
1 2 3 4 5 7 16 19	1518	110101000	111101101000000001001000000010010000000101001100 11	00000	00000	00000
1 2 3 4 5 9 13 16	6072	000001001	010011000010000001110000100000001001010000000011 0	00000	10000	11010
1 2 3 4 7 8 16 22	3036	100000000	000000000000000000000000111111111111111111111111 1	11111	00000	11111
1 2 3 4 5 9 12 17	3036	010000000	101010000110100001000101111010010010010010101	01110	00011	10101
1 2 3 4 5 8 9 16	6072	101000001	110011100000000000111100000111000000001000100001	00011	11111	00000
1 2 3 4 5 8 9 13	6072	001111010	000000000110100010101101000000000100100101000011	00111	00110	00110
1 2 3 4 5 8 12 14	6072	011000110	100001100011000000001000001101010100100001010000 0	00001	01110	10110
1 2 3 4 5 8 9 12	6072	000111101	010001000100110001110101000001010010100001000001	01000	00000	01110
1 2 3 4 5 8 12 21	1518	110101000	111101101000000001001000000010010000000101001100 11	11111	11111	00000
1 2 3 4 7 8 9 17	759	000000000	111111111111111111111111111111111111111111111111 1	00000	11111	00000

4.3 Designs with automorphism group $PSL(2,23)$

TABLE III. **7-(24,8,λ) Designs**

orbits on 8-subsets of V: representative	len.	solutions $\lambda =$ 4 ⊙	5	6	7	8	8 ⊙
f 1 2 3 4 5 6 7 12	6072	1	00000	01101	00000	00000	1110
f 1 2 3 4 5 6 7 8	6072	0	11111	10010	10110	10010	0000
e 1 2 3 4 5 6 7 13	6072	0	01101	11100	10010	11111	0111
e 1 2 3 4 5 7 9 13	6072	0	00000	10001	10111	11001	1011
e 1 2 3 4 5 7 8 13	6072	0	00000	00010	10001	00110	0000
f 1 2 3 4 5 8 10 20	3036	0	11111	00010	01111	11100	0100
1 2 3 4 5 7 10 20	6072	1	00000	00000	01100	01001	
1 2 3 4 5 10 11 14	6072	0	00000	01001	10010	10000	
1 2 3 4 5 8 10 11	6072	0	01100	01001	10010	10111	
e 1 2 3 4 5 7 13 17	6072	0	10011	10100	01101	00011	1011
e 1 2 3 4 5 7 13 19	6072	0	00000	00100	01000	00101	1101
1 2 3 4 5 10 11 16	6072	0	00000	10010	00001	11010	
1 2 3 4 5 6 7 19	6072	0	10010	11101	01000	01111	
f 1 2 3 4 5 6 7 9	6072	0	00000	00000	01000	01101	1110
e 1 2 3 4 5 6 7 10	6072	0	00010	01011	10000	00000	0001
e 1 2 3 4 5 6 7 11	3036	1	01010	10100	10111	11111	1000
e 1 2 3 4 5 6 7 14	6072	0	00000	00010	01001	01000	0011
e 1 2 3 4 5 6 7 15	6072	1	10000	00000	00111	10101	1101
f 1 2 3 4 5 6 7 16	6072	0	00000	00000	00111	11000	0000
1 2 3 4 5 6 7 17	6072	0	01000	00011	01000	10010	
1 2 3 4 5 6 7 18	6072	0	00001	10000	11111	11111	
1 2 3 4 5 6 7 21	3036	0	10101	01011	11001	00111	
1 2 3 4 5 6 7 22	6072	0	00100	00100	00101	00000	
e 1 2 3 4 5 7 11 15	6072	1	10100	00000	01000	00000	0100
e 1 2 3 4 5 7 9 10	6072	0	10011	00000	10010	11010	0110
f 1 2 3 4 5 7 13 16	6072	0	11111	11110	01111	11111	0010
1 2 3 4 5 7 13 21	6072	0	01000	01010	00111	01111	
e 1 2 3 4 5 7 10 15	6072	0	00101	01011	00100	10100	1100
f 1 2 3 4 5 6 8 14	6072	0	00000	00001	00000	00000	1100
f 1 2 3 4 5 7 8 22	6072	1	00000	01100	01000	01100	0010
1 2 3 4 5 9 11 16	6072	0	01101	00001	11110	00010	
f 1 2 3 4 5 6 13 17	3036	1	11110	11111	00000	00000	1100
e 1 2 3 4 5 7 10 11	6072	0	10000	10100	11000	11111	0011
1 2 3 4 5 9 10 11	6072	1	00010	10100	00001	11101	
1 2 3 4 5 7 18 21	6072	0	01011	00010	10110	00000	
e 1 2 3 4 5 6 8 9	6072	0	01000	10001	10001	00100	1011
e 1 2 3 4 5 6 8 10	6072	0	10111	01100	01000	11111	0111
e 1 2 3 4 5 6 8 12	6072	0	01000	10010	00100	00111	0010
f 1 2 3 4 5 6 8 15	3036	1	00000	01111	11110	00000	1100
f 1 2 3 4 5 6 8 16	6072	0	00110	00000	10000	01000	0110
e 1 2 3 4 5 6 8 17	6072	0	00001	01010	00001	00010	1111
e 1 2 3 4 5 6 8 18	3036	1	01100	10110	01010	11001	0000
e 1 2 3 4 5 6 8 21	6072	0	00000	00001	11101	00010	1001
e 1 2 3 4 5 6 8 22	6072	1	10010	11100	00010	11011	0001
f 1 2 3 4 5 6 8 23	3036	0	00001	01111	11111	11111	0000
e 1 2 3 4 5 6 9 10	6072	0	00101	00010	01010	00010	0100
1 2 3 4 5 6 9 12	6072	0	00010	00000	11101	01000	
e 1 2 3 4 5 6 9 13	6072	0	01101	11110	01010	01110	0110
e 1 2 3 4 5 6 9 14	6072	0	01010	00000	00111	10011	1000
1 2 3 4 5 6 9 16	6072	1	10000	10000	00000	10011	
1 2 3 4 5 6 9 17	6072	0	10011	11100	00110	11100	
1 2 3 4 5 6 9 18	6072	0	10100	00001	10000	10000	
e 1 2 3 4 5 6 9 19	6072	0	01000	00001	10100	10111	1111
1 2 3 4 5 6 9 21	6072	1	10001	10011	00011	11111	
f 1 2 3 4 5 6 9 22	3036	1	00000	01111	11111	11111	0000
1 2 3 4 5 6 9 23	6072	0	01101	01110	11101	00000	
f 1 2 3 4 5 6 10 12	6072	0	00000	11100	00010	11000	0100
e 1 2 3 4 5 6 10 13	6072	0	01000	00010	00111	00011	1001
e 1 2 3 4 5 6 10 14	6072	0	00101	00000	10101	11110	0100
1 2 3 4 5 6 10 16	6072	0	10001	10010	00001	00100	
e 1 2 3 4 5 6 10 17	6072	1	00000	00000	00100	10001	1001
e 1 2 3 4 5 6 10 18	6072	0	00000	01010	11011	01010	0010
e 1 2 3 4 5 6 10 19	3036	0	10100	00111	01110	11001	1110
f 1 2 3 4 5 6 10 21	6072	0	11111	10000	01010	00000	0000
1 2 3 4 5 6 10 23	6072	1	00000	00000	11101	11101	
1 2 3 4 5 6 12 13	6072	0	00000	00011	11010	11000	
1 2 3 4 5 6 12 14	6072	1	00000	00100	10100	10000	
1 2 3 4 5 6 12 16	6072	0	01110	01100	00011	01111	
1 2 3 4 5 6 12 17	6072	0	10000	00000	11111	10100	
1 2 3 4 5 6 12 18	6072	0	00010	00010	00100	01101	
1 2 3 4 5 6 12 21	3036	0	01011	01011	01000	11111	
e 1 2 3 4 5 6 13 14	3036	0	10010	11001	11101	00111	1010
1 2 3 4 5 6 13 16	6072	0	00000	10101	10100	11101	
f 1 2 3 4 5 6 13 18	6072	1	00000	00000	10000	00001	0000
1 2 3 4 5 6 13 23	3036	0	10011	11001	01010	00110	
1 2 3 4 5 6 17 18	3036	0	01101	00101	10101	00110	
e 1 2 3 4 5 8 10 13	6072	1	10000	10001	11111	00000	0010
1 2 3 4 5 8 20 21	6072	1	10000	00000	00000	00101	
1 2 3 4 5 10 12 17	3036	0	00110	01111	01101	11010	
1 2 3 4 5 8 14 20	6072	0	01000	00101	01100	01111	
e 1 2 3 4 5 7 8 18	6072	0	10011	01000	00011	00000	0000
1 2 3 4 5 8 10 17	3036	0	00000	01100	10010	00000	
e 1 2 3 4 5 7 10 17	6072	0	01000	10010	10011	00111	1011
1 2 3 4 5 7 16 17	6072	0	10000	10010	10010	01101	
e 1 2 3 4 5 7 9 16	6072	1	01000	00011	11010	00110	0100
1 2 3 4 5 7 10 18	6072	0	01100	10110	01001	00101	
e 1 2 3 4 5 7 9 22	6072	0	01000	00000	00101	00010	0010
e 1 2 3 4 5 9 10 12	3036	0	00111	01100	10100	11100	1000
1 2 3 4 5 8 9 11	6072	1	10000	00001	01100	11001	
e 1 2 3 4 5 7 8 20	3036	0	00000	11100	11100	00001	1001
e 1 2 3 4 5 8 9 10	6072	0	10001	11000	10001	00010	0011
1 2 3 4 5 8 13 20	6072	0	01000	00010	00000	00010	
1 2 3 4 5 10 14 23	3036	0	11111	01100	00000	00000	
f 1 2 3 4 5 7 18 20	6072	0	11110	00001	00001	00111	0000
e 1 2 3 4 5 7 8 10	6072	1	00010	00010	01001	01011	1011
e 1 2 3 4 5 7 8 21	6072	0	00100	10000	11000	00000	0111
e 1 2 3 4 5 7 10 22	6072	0	00000	10000	00000	11001	1100
1 2 3 4 5 8 16 17	6072	0	00000	01001	00101	10000	
1 2 3 4 5 8 11 21	6072	0	00000	00100	11110	11111	
1 2 3 4 5 8 10 21	6072	0	00001	00110	00000	10011	
f 1 2 3 4 5 7 8 17	6072	0	11111	11101	01111	01110	1110
1 2 3 4 5 8 10 16	3036	1	00000	11111	11000	00100	
1 2 3 4 5 9 11 17	3036	1	10101	01100	00000	11011	
e 1 2 3 4 5 7 10 16	6072	0	01100	00001	00000	00010	0111
1 2 3 4 5 7 18 22	6072	0	01110	00001	11110	11010	
e 1 2 3 4 5 7 11 17	6072	0	10010	01001	00001	01000	1010
e 1 2 3 4 5 7 8 11	3036	1	00000	00000	00000	00000	0100
1 2 3 4 5 7 11 16	6072	0	00001	11110	10100	11101	
1 2 3 4 5 7 14 21	6072	0	01100	10100	00000	10000	
e 1 2 3 4 5 7 8 9	6072	0	00000	00110	00011	11100	0100
e 1 2 3 4 5 7 10 14	3036	0	01010	01100	11111	00100	1110
1 2 3 4 5 7 9 19	6072	0	00100	10001	10000	00000	
e 1 2 3 4 5 7 8 16	6072	1	10101	01011	01010	11011	0001
e 1 2 3 4 5 7 8 14	6072	0	00001	01000	00000	11111	0101
e 1 2 3 4 5 7 11 19	6072	0	01100	00001	00011	10110	1110
1 2 3 4 5 7 11 20	6072	0	00010	00000	00011	01000	
f 1 2 3 4 5 7 11 18	6072	0	00000	01101	00010	11111	1110
1 2 3 4 5 7 11 14	6072	0	10011	10001	00010	11111	
1 2 3 4 5 7 17 21	3036	0	00000	00000	11111	11111	
e 1 2 3 4 5 7 9 21	6072	1	00000	01111	01100	11100	1101
e 1 2 3 4 5 7 9 17	6072	1	00000	11001	10000	10101	1001
e 1 2 3 4 5 7 16 19	759	0	10011	11100	00000	00000	1000
e 1 2 3 4 5 7 14 20	3036	1	11110	11111	11111	11111	1001
1 2 3 4 5 7 20 22	6072	0	00000	00010	11010	10000	
1 2 3 4 5 7 21 22	6072	0	10011	10000	01001	00101	
e 1 2 3 4 5 7 9 18	6072	0	10111	10000	00100	00011	1101
e 1 2 3 4 5 7 9 14	3036	1	00000	01100	00000	11011	0010
1 2 3 4 5 14 17 21	3036	1	00110	00000	11000	11111	
e 1 2 3 4 7 8 16 22	1518	0	00001	11110	10011	00000	1100
1 2 3 4 5 8 9 17	6072	0	01010	00110	10100	00101	
f 1 2 3 4 5 9 12 17	3036	1	00001	10001	11000	11011	0000
f 1 2 3 4 5 8 9 16	6072	0	00000	00010	01111	01100	0100
1 2 3 4 7 9 10 12	1518	1	00000	11101	01101	00000	
e 1 2 3 4 5 9 13 16	3036	0	00110	00000	00000	00000	1111
e 1 2 3 4 5 8 9 15	3036	0	00000	11111	11111	11111	1000
1 2 3 4 5 8 14 21	3036	1	01100	11111	00000	00000	
e 1 2 3 4 5 8 9 12	3036	0	10011	01100	11000	00000	0101
1 2 3 4 5 8 12 14	3036	0	10010	11111	11111	11111	
1 2 3 4 5 8 13 16	3036	0	00001	01100	00000	00000	
1 2 3 4 5 8 16 21	3036	1	01100	01100	00111	11111	
1 2 3 4 5 11 14 23	759	1	01100	11100	00000	00000	
f 1 2 3 4 5 8 12 21	1518	0	00001	00000	00001	00000	1000
f 1 2 3 4 7 8 9 17	759	1	00000	00000	11111	00000	0000

The last column in the table shows all 7-(24,8,8) designs with automorphism group $PGL(2,23)$. Orbit representatives preceded by "f" represent an orbit of

$PGL(2,23)$ which is also an orbit of $PSL(2,23)$. Orbit representatives preceded by "s" represent an orbit of $PGL(2,23)$ which splits into 2 orbits of $PSL(2,23)$.

4.4 Designs with automorphism group $PGL(2,23)p$

TABLE IV. 6-(25,8,λ) Designs

orb. on 8-subs. of V		solutions λ =						orb. on 8-subs. of V		solutions λ =					
representative	len.	36	45	54	63	72	81	representative	len.	36	45	54	63	72	81
2 3 4 5 6 7 8 13	6072	00001	00000	11000	11000	10000	00000	2 3 4 5 6 7 9 18	3036	00000	11111	00000	11111	00000	11111
1 2 3 4 5 6 11 21	6072	00001	01101	11011	00100	11010	11010	2 3 4 5 6 7 9 19	6072	01101	01111	00001	11111	00000	11010
2 3 4 5 6 7 8 9	12144	00000	10010	00000	00001	01011	11111	2 3 4 5 6 7 9 22	12144	00000	11010	00000	11001	11110	11111
2 3 4 5 6 7 8 14	6072	10000	01001	00011	00011	00100	00111	2 3 4 5 6 7 9 23	12144	10100	00001	11111	00000	11111	11111
2 3 4 5 6 8 10 14	12144	00000	00000	00000	00000	00000	00101	2 3 4 5 6 7 9 24	12144	00011	11111	00000	11111	00000	00000
2 3 4 5 6 8 9 14	6072	10110	01111	11111	00101	10111	11100	2 3 4 5 6 7 10 11	12144	00000	00000	00000	00000	00000	00100
2 3 4 5 6 9 11 21	12144	10010	00000	00000	00010	01101	00001	2 3 4 5 6 7 10 14	6072	00000	11111	11111	00000	11111	11111
2 3 4 5 6 8 14 20	12144	01100	00000	00100	00110	10111	11010	2 3 4 5 6 7 10 15	6072	00000	00000	11111	00000	00000	00000
2 3 4 5 6 8 14 18	12144	10010	11111	00000	11000	11000	01111	2 3 4 5 6 7 10 20	6072	00000	11111	00000	11111	11111	11111
1 2 3 4 5 6 7 8	12144	01100	00000	01011	10110	10101	00100	2 3 4 5 6 7 10 23	3036	11111	11111	00000	00000	00000	00000
1 2 3 4 5 6 8 14	12144	01000	00000	00100	00000	01100	11000	2 3 4 5 6 7 11 13	6072	00000	00000	00000	00000	11111	11111
1 2 3 4 5 6 14 18	12144	00000	10100	11100	11000	01001	11000	2 3 4 5 6 7 11 14	12144	00000	00000	11111	00000	11111	11111
2 3 4 5 6 7 8 10	12144	00010	10100	00000	10000	10001	11000	2 3 4 5 6 7 11 15	12144	01011	00000	00000	00000	00000	00000
2 3 4 5 6 7 8 11	6072	00010	00000	10001	01001	00010	10110	2 3 4 5 6 7 11 16	6072	00000	00000	00000	00000	00000	00000
2 3 4 5 6 7 8 12	6072	01100	00010	00001	00101	00011	00101	2 3 4 5 6 7 11 19	12144	00000	00000	00000	00000	00000	11111
2 3 4 5 6 7 8 15	12144	00100	11001	10111	01111	00011	00011	2 3 4 5 6 7 11 20	12144	00000	11100	00000	00001	10010	11110
2 3 4 5 6 7 8 16	12144	00000	00100	00110	11110	00100	00100	2 3 4 5 6 7 11 22	12144	00010	10101	00000	00000	00100	10000
2 3 4 5 6 7 8 17	12144	00000	00001	01000	11011	00010	00011	2 3 4 5 6 7 14 15	6072	10111	00010	10000	11111	01010	00000
2 3 4 5 6 8 12 14	6072	10000	01101	10110	00001	01100	10011	2 3 4 5 6 7 14 19	12144	11001	00010	01000	00001	11100	11011
2 3 4 5 6 8 10 11	12144	00000	00000	00000	01000	00000	11000	2 3 4 5 6 9 11 14	6072	00000	00000	00000	00000	00000	00000
2 3 4 5 6 8 14 17	12144	00000	00010	00001	00101	11110	10110	2 3 4 5 6 8 10 17	12144	10011	00000	01000	00000	01001	11000
2 3 4 5 6 8 11 12	6072	01001	00000	00001	10000	01010	11100	2 3 4 5 6 10 11 13	6072	10000	01010	11100	11000	11110	01000
2 3 4 5 6 8 11 14	6072	11100	00010	10001	11010	00010	00110	2 3 4 5 6 9 10 11	12144	00000	00000	00001	00000	11000	01111
2 3 4 5 6 7 9 15	6072	00001	10010	11110	11001	11101	00111	2 3 4 5 6 8 9 19	12144	01010	00000	00000	00000	00111	10100
2 3 4 5 6 8 9 23	12144	00001	00000	01100	00100	10001	11001	2 3 4 5 6 8 9 21	6072	10011	00111	00000	00000	00000	00000
2 3 4 5 6 7 14 18	12144	10111	00010	11000	00010	00011	01000	2 3 4 5 6 8 11 18	12144	10100	00000	00110	00110	00111	11101
1 2 3 4 5 6 7 9	12144	00000	01010	10010	11100	10100	10101	2 3 4 5 6 8 10 23	12144	01100	10100	00101	00011	00000	00100
1 2 3 4 5 6 7 10	12144	01000	00101	00101	00100	11111	00000	2 3 4 5 6 8 15 21	12144	00010	10000	10000	00110	01000	00011
1 2 3 4 5 6 7 11	6072	00100	11000	11010	10011	00000	00111	2 3 4 5 6 8 19 21	12144	10100	01100	10110	11111	10001	01111
1 2 3 4 5 6 7 14	6072	00010	00001	00011	01000	10100	01001	2 3 4 5 6 8 9 11	12144	01001	01001	01111	11100	10101	00000
1 2 3 4 5 6 10 11	12144	00000	00000	00000	00001	01010	00111	2 3 4 5 6 8 9 23	12144	11000	00000	11111	00000	11111	00000
1 2 3 4 5 6 8 21	6072	00001	01000	11010	11111	01100	00011	2 3 4 5 6 8 11 23	12144	00101	00100	10100	00100	00000	11010
1 2 3 4 5 6 9 11	6072	01011	11110	00100	11111	10110	11000	2 3 4 5 6 8 9 10	6072	00000	11111	00000	11111	00000	11111
1 2 3 4 5 6 8 15	12144	10001	11001	01011	00011	11000	00010	2 3 4 5 6 8 9 15	12144	00000	00000	00000	00000	00000	11111
1 2 3 4 5 6 8 18	12144	00000	00000	00000	00000	00000	11111	2 3 4 5 6 8 10 18	12144	00000	00000	00000	00000	11111	00000
1 2 3 4 5 6 11 15	6072	00000	00100	10000	00000	01111	11110	2 3 4 5 6 8 9 18	12144	00000	00000	01010	00011	11011	00011
1 2 3 4 5 6 11 18	6072	00100	00001	00111	11110	11100	11111	2 3 4 5 6 8 10 15	12144	00100	00001	11111	11111	11111	00000
1 2 3 4 5 6 8 9	6072	00001	01000	00010	00101	11001	11000	2 3 4 5 6 8 11 15	12144	00000	00011	01011	11010	10101	00111
1 2 3 4 5 6 8 11	12144	00000	00000	00001	11001	10100	00111	2 3 4 5 6 8 11 17	6072	00000	00001	00011	00100	01110	10111
1 2 3 4 5 6 8 10	6072	11111	00000	00000	11111	00000	00000	2 3 4 5 6 8 10 19	12144	00011	10110	00000	00000	00000	11111
1 2 3 4 5 6 11 14	6072	10000	10110	00100	00000	00000	00010	2 3 4 5 6 8 12 18	6072	10100	10101	00101	00011	01111	10000
1 2 3 4 5 6 8 12	12144	00000	00000	11010	00000	00000	01001	2 3 4 5 6 8 9 12	12144	00000	11111	11111	11111	11111	11111
1 2 3 4 5 6 8 17	12144	00000	00000	00000	00000	00000	00000	2 3 4 5 6 8 10 22	12144	00000	01000	00000	11111	11111	00000
1 2 3 4 5 6 8 19	12144	00010	10010	01000	00010	00011	10101	2 3 4 5 6 8 9 17	6072	00011	11000	11111	11111	11111	11111
1 2 3 4 5 6 8 23	6072	00000	00000	00000	00000	11111	00000	2 3 4 5 6 8 12 20	1518	00000	11111	11111	11111	00000	11111
1 2 3 4 5 6 8 16	12144	01000	00100	00100	00010	00111	01110	2 3 4 5 6 8 12 19	6072	01000	11000	00010	00000	10001	00100
1 2 3 4 5 6 9 17	12144	00000	10000	11001	11001	11111	10110	2 3 4 5 6 8 17 20	12144	10000	01010	10000	11100	00000	01000
1 2 3 4 5 6 10 18	12144	00000	01110	10101	00101	10110	00000	2 3 4 5 6 10 14 17	6072	00111	00000	00000	00000	11111	00000
1 2 3 4 5 6 10 17	6072	11011	00101	00111	00110	01001	00000	2 3 4 5 6 9 17 23	6072	00100	00111	00000	11111	00000	00000
1 2 3 4 5 6 10 15	3036	11111	11111	00000	00000	00000	00000	2 3 4 5 6 10 15 18	6072	01100	00001	01101	11001	00011	10000
1 2 3 4 5 6 14 22	6072	10000	01000	00010	00100	10100	11011	2 3 4 5 6 9 10 17	12144	00000	00000	00011	00110	00000	00000
1 2 3 4 5 6 14 18	12144	00011	00000	01000	11001	01000	00001	2 3 4 5 6 9 10 14	3036	01111	11111	00000	11111	00000	11111
1 2 3 4 5 6 10 14	6072	00000	00000	11111	11111	00000	11111	2 3 4 5 6 9 15 18	6072	01000	00000	00000	11111	00000	00000
1 2 3 4 5 6 9 15	12144	11100	00001	10000	11010	11011	01010	2 3 4 5 6 9 10 13	1518	00000	00000	00000	11111	00000	00000
1 2 3 4 5 6 9 14	12144	00100	00000	00000	00000	00000	11000	1 2 3 4 5 6 9 10	6072	00000	00000	00000	00000	00000	11111
2 3 4 5 6 7 9 10	3036	00000	11111	00000	11111	00000	11111	1 2 3 4 5 6 9 13	3036	10000	00000	00000	00000	00000	11111
2 3 4 5 6 7 9 11	12144	01000	11000	11100	00010	01011	11111	1 2 3 4 5 6 9 10	6072	00000	00000	00000	00000	00000	11111
2 3 4 5 6 7 9 13	12144	00000	00000	10110	11101	01110	10101	2 3 4 5 6 9 13 22	759	00000	11111	00000	11111	00000	11111
2 3 4 5 6 7 9 16	12144	00000	00010	11011	11000	10000	01011	2 3 4 5 6 9 10 18	3036	00000	00000	00000	11111	00000	00000
2 3 4 5 6 7 9 17	12144	00000	00001	00000	00100	01111	10101								

4.5 Designs with automorphism group $P\Gamma L(2,25)$

TABLE V. 7-(26,9,λ) Designs

orbits on 9-subsets of V		solutions		
representative	length	λ = 54	λ = 63	λ = 81
1 2 3 4 5 7 11 17 18	15600	11111	11000	00000
1 2 3 4 5 6 7 8 9	31200	10001	01000	11011
1 2 3 4 5 7 8 10 24	31200	01000	10011	11100
1 2 3 4 5 7 9 11 17	31200	10100	10010	11110
1 2 3 4 5 7 11 17 26	31200	00110	01011	00011
1 2 3 4 5 7 8 10 17	31200	00010	00000	10100
1 2 3 4 5 7 8 11 17	31200	01000	01000	01000
1 2 3 4 5 7 8 11 18	31200	00011	00101	01001
1 2 3 4 5 7 8 18 23	31200	01000	00100	00000
1 2 3 4 5 7 8 10 20	15600	00000	00000	00000
1 2 3 4 5 7 8 11 21	31200	00000	01101	00110
1 2 3 4 5 6 7 8 22	31200	00001	00101	10111
1 2 3 4 5 6 7 8 15	31200	01110	00011	01101
1 2 3 4 5 6 7 8 10	31200	10000	00110	00111
1 2 3 4 5 6 7 8 21	15600	00000	11111	11111
1 2 3 4 5 6 7 8 12	31200	00000	11011	11000
1 2 3 4 5 6 7 8 13	7800	00000	00000	11111
1 2 3 4 5 7 9 14 17	31200	00000	10100	01111
1 2 3 4 5 7 8 23 26	31200	00110	00000	11001
1 2 3 4 5 7 9 11 23	15600	10111	10000	11111
1 2 3 4 5 7 11 25 26	31200	00000	01010	11110
1 2 3 4 5 7 8 21 22	31200	10110	00100	10111
1 2 3 4 5 7 8 12 22	15600	01001	00001	00000
1 2 3 4 5 7 8 11 26	31200	00001	10101	00011
1 2 3 4 5 7 8 11 25	15600	10110	00001	11111
1 2 3 4 5 7 8 15 17	15600	00000	11000	11111
1 2 3 4 5 7 8 25 26	31200	01101	10010	01010
1 2 3 4 5 7 8 12 20	15600	00000	11111	11111
1 2 3 4 5 7 8 13 21	7800	00000	11111	00000
1 2 3 4 5 7 8 15 20	15600	11010	10100	11001
1 2 3 4 5 7 8 16 22	15600	11111	00000	00000
1 2 3 4 5 7 9 11 14	31200	10110	00000	00000
1 2 3 4 5 7 9 14 18	31200	10000	01000	10101
1 2 3 4 5 7 9 14 23	31200	11000	00000	00110
1 2 3 4 5 7 8 9 17	15600	10001	10000	11111
1 2 3 4 5 7 8 11 14	31200	00111	00010	00000
1 2 3 4 5 7 8 11 18	31200	11110	11011	00000
1 2 3 4 5 7 8 9 13	15600	00000	00000	11111
1 2 3 4 5 7 9 14 20	31200	00000	11101	11101
1 2 3 4 5 7 8 15 25	31200	00011	01101	00000
1 2 3 4 5 7 8 17 21	15600	00000	00000	00000
1 2 3 4 5 7 8 9 14	31200	00110	00100	00001
1 2 3 4 5 7 8 12 21	31200	01001	01101	00000
1 2 3 4 5 7 9 11 20	7800	00000	00000	11111
1 2 3 4 5 7 8 9 11	15600	11100	11101	00101
1 2 3 4 5 7 9 11 22	31200	00001	01010	10101
1 2 3 4 5 7 8 9 15	31200	10001	11010	00010
1 2 3 4 5 7 8 13 24	31200	00010	10001	01100
1 2 3 4 5 7 8 12 17	15600	10001	01000	00000
1 2 3 4 5 7 8 10 14	31200	00001	00010	01011
1 2 3 4 5 7 8 13 26	31200	00000	00110	11111
1 2 3 4 5 7 10 18 23	15600	10000	00110	11111
1 2 3 4 5 7 8 10 11	31200	00110	01111	10101
1 2 3 4 5 7 8 11 22	31200	01000	11011	11110
1 2 3 4 5 7 8 11 24	31200	00100	00010	10011
1 2 3 4 5 7 8 11 12	31200	00000	00010	00000
1 2 3 4 5 7 8 9 20	5200	00000	11111	00000
1 2 3 4 5 7 8 11 23	31200	11100	10001	10110
1 2 3 4 5 7 10 16 18	31200	00010	00000	10100
1 2 3 4 5 7 8 10 16	15600	00000	11000	11111
1 2 3 4 5 7 8 10 13	15600	01000	10111	00000
1 2 3 4 5 6 7 9 23	31200	11000	00010	01111
1 2 3 4 5 6 7 9 18	31200	11110	00100	01000
1 2 3 4 5 6 7 9 16	31200	00111	01010	10101
1 2 3 4 5 6 7 14 20	31200	00100	00100	01001
1 2 3 4 5 7 8 9 23	15600	00000	01110	11100
1 2 3 4 5 7 8 9 26	15600	00011	10100	11001
1 2 3 4 5 7 8 15 26	31200	00110	00010	00010
1 2 3 4 5 7 8 15 18	31200	11110	10001	01111
1 2 3 4 5 7 8 16 26	15600	11111	00000	00000
1 2 3 4 5 7 8 12 26	31200	01000	10111	00000
1 2 3 4 5 7 8 9 22	31200	00001	01000	01111
1 2 3 4 5 7 8 22 26	31200	00001	01010	10001
1 2 3 4 5 7 8 15 22	31200	10000	10001	00110
1 2 3 4 5 7 8 10 26	31200	00100	01010	10000
1 2 3 4 5 7 8 24 26	31200	00000	00000	10001
1 2 3 4 5 7 8 21 26	31200	11000	10011	01000
1 2 3 4 5 7 10 24 26	31200	10101	00100	11111
1 2 3 4 5 7 9 16 25	31200	01000	10100	11111
1 2 3 4 5 7 8 10 21	31200	00010	00110	11111
1 2 3 4 5 7 8 12 16	31200	01000	01001	00001
1 2 3 4 5 7 8 12 25	31200	01110	10101	00000
1 2 3 4 5 7 8 9 25	15600	00000	00000	11111
1 2 3 4 5 7 8 15 25	31200	00001	10100	01010
1 2 3 4 5 7 8 23 25	31200	00001	11100	10010
1 2 3 4 5 7 9 16 18	15600	00000	00000	00000
1 2 3 4 5 7 8 9 18	31200	00001	10010	10100
1 2 3 4 5 7 8 21 24	31200	11000	10000	11010
1 2 3 4 5 7 9 18 25	31200	11000	11100	00000
1 2 3 4 5 7 8 10 12	15600	00111	11111	11111
1 2 3 4 5 7 8 18 25	31200	01000	01000	11101
1 2 3 4 5 7 8 10 23	31200	00000	11001	10110
1 2 3 4 5 7 8 16 25	15600	10111	01000	00000
1 2 3 4 5 7 14 20 22	15600	00001	00000	00000
1 2 3 4 5 7 8 10 25	31200	00000	00000	11111
1 2 3 4 5 7 8 15 21	31200	01000	00100	11001
1 2 3 4 5 7 8 12 16	15600	11010	00101	11010
1 2 3 4 5 7 8 9 21	15600	10000	11001	00000
1 2 3 4 5 7 8 9 12	31200	00001	00001	00010
1 2 3 4 5 7 8 12 14	3900	11111	11111	00000
1 2 3 4 5 7 9 20 24	31200	00001	00000	11111
1 2 3 4 5 7 8 24 25	31200	00000	11101	01010
1 2 3 4 5 7 9 16 20	15600	00000	00000	00000
1 2 3 4 5 7 8 15 16	31200	10100	00110	00000
1 2 3 4 5 7 8 15 24	15600	00001	00000	11111
1 2 3 4 5 7 8 9 24	31200	10011	00000	01100
1 2 3 4 5 7 8 9 16	31200	01000	11101	00000
1 2 3 4 5 7 9 16 22	7800	00000	00000	11111
1 2 3 4 5 7 8 9 10	31200	00100	01000	00111
1 2 3 4 5 7 8 9 19	31200	00111	00011	10001
1 2 3 4 5 7 8 10 15	7800	11111	00000	00000
1 2 3 4 5 7 8 10 16	15600	11111	00000	11111
1 2 3 4 5 7 8 16 25	15600	01000	01111	11111
1 2 3 4 7 8 10 12 15	31200	11010	10010	01010
1 2 3 4 7 8 10 12 13	7800	00000	00000	00000
1 2 3 4 7 8 9 12 19	15600	11111	11111	00000
1 2 3 4 7 8 9 17 19	31200	10000	00001	11100
1 2 3 4 7 8 11 13 26	3900	00000	11111	11111
1 2 3 4 7 8 11 13 14	5200	00000	11111	00000
1 2 3 4 7 8 11 14 15	15600	00111	00000	11111
1 2 3 4 7 8 11 12 20	5200	00000	11111	00000
1 2 3 4 7 8 11 13 26	7800	11111	11111	11111
1 2 3 4 7 8 9 16 19	15600	00000	11111	11111
1 2 3 4 7 8 9 14 17	15600	00000	11111	00000
1 2 3 4 7 8 9 12 16	31200	01001	00000	00000
1 2 3 4 7 8 11 12 13	15600	00000	01001	00011
1 2 3 4 7 8 10 15 16	15600	11111	11111	00000
1 2 3 4 7 8 9 14 22	7800	00000	00000	11111
1 2 3 4 7 8 14 15 22	31200	10110	00000	11111
1 2 3 4 7 8 9 14 21	7800	00000	00000	00000
1 2 3 4 7 8 9 14 16	2600	00000	11111	00000
1 2 3 4 7 8 15 22 23	1950	00000	11111	11111

4.6 Designs with automorphism group $PGL(2, 25)$

TABLE VI. 7-(26,8,6) Designs

orbits on 8-subsets of V		solutions	orbits on 8-subsets of V		solutions
representative	length	$\lambda = 6$ δ	representative	length	$\lambda = 6$ δ
1 2 3 4 5 7 11 17	7800	00111001111111	1 2 3 4 7 11 14 26	15600	10011010101010
1 2 3 4 5 6 7 8	15600	10000000100010	1 2 3 4 7 8 12 26	7800	01001001111001
1 2 3 4 5 7 11 26	15600	00111111001100	1 2 3 4 7 8 11 14	2600	00000000000000
1 2 3 4 5 7 9 14	15600	01010101000001	1 2 3 4 7 8 21 22	7800	10000110110110
1 2 3 4 5 7 9 11	15600	00000000000011	1 2 3 4 7 8 10 12	7800	00101101100000
1 2 3 4 5 7 11 15	15600	01001000000000	1 2 3 4 7 8 10 13	1950	11111111111111
1 2 3 4 5 7 11 25	15600	10000010010100	1 2 3 4 7 8 12 20	7800	11011000100101
1 2 3 4 5 7 11 20	15600	10000100101000	1 2 3 4 7 8 14 17	15600	00100010000011
1 2 3 4 5 7 8 17	15600	01100010010000	1 2 3 4 7 8 17 26	15600	01101101000010
1 2 3 4 5 7 8 11	3900	00000000000000	1 2 3 4 7 8 9 16	7800	10000110110000
1 2 3 4 5 7 10 11	7800	11100100011010	1 2 3 4 7 8 15 20	7800	00110110111111
1 2 3 4 5 7 11 13	15600	11000000000000	1 2 3 4 7 8 14 23	15600	10010101001010
1 2 3 4 5 7 10 16	7800	00000101000010	1 2 3 4 7 8 13 20	15600	01001000011110
1 2 3 4 5 7 10 20	15600	00010110011100	1 2 3 4 7 8 14 19	15600	10011110000000
1 2 3 4 5 7 8 20	15600	10000100101101	1 2 3 4 7 8 10 17	7800	00010000001001
1 2 3 4 5 7 8 13	15600	00010001000011	1 2 3 4 7 8 19 20	15600	10100000101100
1 2 3 4 5 7 11 19	15600	10011110000001	1 2 3 4 7 14 16 17	15600	01001000010100
1 2 3 4 5 7 11 22	15600	00000000000000	1 2 3 4 7 8 11 13	15600	00001011101001
1 2 3 4 5 6 7 26	15600	00000000000000	1 2 3 4 7 8 13 17	3900	00000000100010
1 2 3 4 5 6 7 14	15600	01000000010001	1 2 3 4 7 8 13 26	15600	10000100000000
1 2 3 4 5 6 7 9	15600	01101010000101	1 2 3 4 7 8 13 21	7800	00000000000000
1 2 3 4 5 7 9 23	7800	00100000000110	1 2 3 4 7 8 21 26	7800	00001100000000
1 2 3 4 5 7 15 16	15600	01101101000000	1 2 3 4 7 8 12 23	7800	00000000000000
1 2 3 4 5 7 13 21	15600	00000010011100	1 2 3 4 8 11 12 13	15600	01010000011000
1 2 3 4 5 7 8 22	15600	00010010100000	1 2 3 4 7 8 15 17	15600	01101001000111
1 2 3 4 5 7 8 26	15600	10111011101000	1 2 3 4 8 13 17 19	3900	00000000000000
1 2 3 4 5 7 10 24	7800	01001001110000	1 2 3 4 6 15 21 22	15600	10000001100010
1 2 3 4 5 7 13 26	15600	00000000000000	1 2 3 4 7 8 10 16	15600	10010110001011
1 2 3 4 5 7 14 15	7800	00001010000001	1 2 3 4 7 8 10 18	15600	01100101010101
1 2 3 4 5 7 14 23	7800	00000000000000	1 2 3 4 7 8 9 17	15600	10100000100100
1 2 3 4 5 7 14 20	15600	01010000011100	1 2 3 4 7 8 15 19	15600	01000010010001
1 2 3 4 5 7 13 22	15600	10010001100000	1 2 3 4 7 8 10 26	2600	00000000000000
1 2 3 4 5 7 8 12	15600	10000100000000	1 2 3 4 7 8 11 15	15600	00000000111100
1 2 3 4 5 7 10 14	15600	00000111010110	1 2 3 4 7 8 15 16	7800	01000101011110
1 2 3 4 5 7 10 19	15600	01110111010100	1 2 3 4 7 8 12 16	15600	00110011111100
1 2 3 4 5 7 9 18	15600	01001000000000	1 2 3 4 7 8 14 15	15600	00110000000000
1 2 3 4 5 7 9 20	15600	00000000000011	1 2 3 4 7 8 14 24	7800	10001010101101
1 2 3 4 5 7 9 16	15600	01000000100011	1 2 3 4 7 8 14 18	15600	01000001101000
1 2 3 4 5 7 8 9	3900	00000000010001	1 2 3 4 7 8 9 14	15600	00010001000001
1 2 3 4 5 7 8 24	15600	10101010000010	1 2 3 4 7 8 14 22	7800	01000100100110
1 2 3 4 5 7 9 13	15600	00010101101000	1 2 3 4 7 8 14 21	15600	10000010100010
1 2 3 4 5 7 15 20	7800	10001110111110	1 2 3 4 7 8 15 22	7800	11111111111111
1 2 3 4 5 7 10 22	15600	10000000010011	1 2 3 4 7 8 22 23	15600	00111100001100
1 2 3 4 5 7 8 15	7800	00001111000011	1 2 3 4 7 9 21 25	15600	01011010000011
1 2 3 4 5 7 10 15	15600	00101010010100	1 2 3 4 7 8 15 23	7800	10001000011001
1 2 3 4 5 7 8 19	15600	00100000000101	1 2 3 4 7 14 15 17	7800	00000000000000
1 2 3 4 5 7 8 21	7800	00000000000000	1 2 3 4 7 8 10 19	15600	00000000010000
1 2 3 4 5 7 16 25	15600	00010000001010	1 2 3 4 7 8 9 12	15600	00100010000010
1 2 3 4 5 7 8 25	15600	00100001010000	1 2 3 4 7 8 18 20	7800	00000000000000
1 2 3 4 5 7 10 25	7800	11110000111100	1 2 3 4 7 8 23 24	3900	11001100110000
1 2 3 4 5 7 13 19	7800	01001101111101	1 2 3 4 7 8 15 21	15600	01000101000010
1 2 3 4 5 7 16 20	7800	01011110110001	1 2 3 4 7 8 10 15	7800	01100100010100
1 2 3 4 5 7 13 20	15600	01101000000010	1 2 3 4 7 9 11 14	3900	11111111011101
1 2 3 4 5 7 10 18	15600	00000001101100	1 2 3 4 7 8 9 20	7800	10011000101000
1 2 3 4 5 7 8 23	15600	01000001011000	1 2 3 4 7 9 11 25	15600	10001010000001
1 2 3 4 5 7 8 16	15600	10010100000001	1 2 3 4 7 14 17 21	7800	00011110010000
1 2 3 4 5 7 13 18	15600	00101001101100	1 2 3 4 7 8 10 20	7800	11000001000010
1 2 3 4 5 7 10 23	15600	00110000000000	1 2 3 4 7 8 21 23	15600	00110000000000
1 2 3 4 5 7 8 10	15600	10000010100100	1 2 3 4 7 8 9 18	7800	00011110110110
1 2 3 4 5 7 8 18	7800	10101101110010	1 2 3 4 7 9 18 22	7800	00000000000000
1 2 3 4 5 7 10 13	15600	00000000100000	1 2 3 4 7 8 20 21	7800	00101101111001
1 2 3 4 7 8 17 24	15600	01000001010001	1 2 3 4 7 8 22 26	975	00000000000000
1 2 3 4 7 11 20 26	7800	11000010000001	1 2 3 4 7 8 11 12	1300	00000000000000
1 2 3 4 7 8 13 14	15600	00110000001111	1 2 3 4 7 8 23 25	3900	11111111101110
1 2 3 4 7 8 11 20	15600	10100101000011	1 2 3 4 7 10 11 17	650	00000000000000
1 2 3 4 7 8 12 24	3900	11001100110000			

These 14 solutions fall into 7 isomorphism classes, since the normalizer $P\Gamma L(2, 25)$ of $PGL(2, 25)$ has orbits of length 2 on the set of these solutions. In fact, δ represents the non-trivial coset of $PGL(2, 25)$ in $P\Gamma L(2, 25)$ and maps solution $2 \times i$ onto solution $2 \times i + 1$ for $i = 0, 1, \ldots, 7$ in the order the solutions are listed above.

The 7-(27, 9, 60) designs are not constructed directly by solving the system of equations given by a Kramer-Mesner matrix. Instead we apply an idea of Tran van Trung [26], to obtain from a t-(v, k, λ) design and a t-$(v, k+1, \lambda(\frac{v-t+1}{k-t+1} - 1))$ design a t-$(v+1, k+1, \lambda\frac{v-t+1}{k-t+1})$ design. Tran van Trung adds an additional point to the base set V and also to each block of the t-(v, k, λ) design and then adds all blocks of the t-$(v, k+1, \lambda(\frac{v-t+1}{k-t+1} - 1))$ design. By this method we obtain from our 7-(26, 8, 6) and 7-(26, 9, 54) designs 7-(27, 9, 60) designs. Also, from our 7-(24, 8, 5) and 7-(24, 9, 40) designs we get 7-(25, 9, 45) designs, from 7-(24, 8, 6) and 7-(24, 9, 48) designs we get 7-(25, 9, 54) designs, and from 7-(24, 8, 8) and 7-(24, 9, 64) designs we get 7-(25, 9, 72) designs. Remarkably, there was no counterpart 7-(24, 9, 56) for 7-(24, 8, 7) to apply Tran van Trung's construction.

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