Dr. A. Betten Spring 2008

MATH 369 Linear Algebra

Midterm # 2

Problem # 1
Consider the linear map $L : \mathbb{R}^3 \to \mathbb{R}^1$, $L(x, y, z) = x + y + z$. Compute a basis for the nullspace.

Problem # 2
Let A be a nonsingular matrix with eigenvalue λ. Show that λ^{-1} is an eigenvalue of A^{-1}.

Problem # 3
Compute the eigenvalues and eigenvectors of the following matrix:

$$\begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix}$$

Problem # 4
Compute the determinant of the following matrix:

$$\begin{pmatrix} y+z & x+z & x+y \\ x & y & z \\ 1 & 1 & 1 \end{pmatrix}$$

Problem # 5
Are the following vectors dependent of independent?
a)

$$\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix},$$

3)

$$\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} -5 \\ 8 \\ -7 \end{pmatrix}, \begin{pmatrix} -7 \\ 13 \\ -5 \end{pmatrix}.$$

Problem # 6
Compute a basis for the solution space of the following system

$$\begin{array}{rrrrrcl} 2x_1 & +3x_2 & +x_3 & & +x_5 & = & 0 \\ -3x_1 & +x_2 & & +x_4 & +2x_5 & = & 0 \end{array}$$

Problem # 7
Express the vector $\vec{v}$ in terms of the basis $\vec{b}_1, \vec{b}_2, \vec{b}_3$ of $\mathbb{R}^3$.

$$\vec{v} = \begin{pmatrix} -27 \\ 1 \\ 2 \end{pmatrix}, \vec{b}_1 = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}, \vec{b}_2 = \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}, \vec{b}_3 = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix},$$

Problem # 8
What is the rank of the following linear map:

$$L : \mathbb{R}^4 \to \mathbb{R}^3,\ L(x, y, z, w) = (2x + 3y, 3y + 4z, 4z + 5w)$$

Determine the nullity (a.k.a. dimension of the kernel).

Problem # 9
Let $B = (\vec{b}_1, \ldots, \vec{b}_n)$ be an ordered basis for $\mathbb{R}^n$. Consider the map $\Phi : \mathbb{R}^n \mapsto \mathbb{R}^n$ $\Phi(\vec{x}) = [\vec{x}]_B$ the coordinate vector of $\vec{x}$ w.r.t the basis B. Show that Φ is a linear map.

Problem # 10
Consider the linear maps $L(x, y, z) = (x + 2y, y + 2z, z + 2x)$ and $M(a, b, c) = (a + c, b + a, b + c)$.
a) Compute the matrix of L
b) Compute the matrix of M
c) Describe the effect of the map $M \circ L$ (i.e. $M(L(x, y, z))$)
d) Compute the matrix of $M \circ L$.
e) How are the matrices in a), b) and d) related?