Dr. A. Betten Spring 2008

MATH 369 Linear Algebra

Assignment # 7

Problem # 39
pg 310. 1acei.

Problem # 40
pg 311. 13. (look up the definition of tr on pg. 309).

Problem # 41
Which of the following functions $\mathbb{R}^2 \to \mathbb{R}^3$ are linear?
a) $(x, y) \mapsto (0, x - y, x + y)$
b) $(x, y) \mapsto (xy, 0, (x_1)(y + 1))$
c) $(x, y) \mapsto (x + 1, y + 1, 3)$
d) $(x, y) \mapsto (-y, -x, 3x - 7y)$

Problem # 42
Let $A = \{1, \cos(2x), \cos^2(x), \sin^2(x)\}$ as elements of $C[-2\pi, 2\pi]$. Determine the dimension of $\operatorname{Span}(A)$.

Problem # 43
Let $p(x) = \frac{1}{2}(x^2 - 5x + 6)$, $q(x) = -x^2 + 4x - 3$, $r(x) = \frac{1}{2}(x^2 - 3x + 2)$.
a) Show that $B = \{p(x), q(x), r(x)\}$ is a basis of P_3 (the space of quadratic polynomials).
b) For each of $1, 1 + x, x + x^2$, find coefficients a, b, c such that $a \cdot p(x) + b \cdot q(x) + c \cdot r(x)$ equals the given term.
c) Determine a polynomial $f(x)$ such that $f(1) = 5, f(2) = -1, f(3) - 4$.

Problem # 44
Is there a linear map $\mathbb{R}^2 \to \mathbb{R}^2$ that maps the vectors $\vec{x} = (1, 0)^\top$, $\vec{y} = (1, 2)^\top$ and $\vec{z} = (3, 4)^\top$ as described below?
a) $\vec{x} \mapsto (2, 3)^\top$, $\vec{y} \mapsto (0, 1)^\top$, $\vec{z} \mapsto (1, 5)^\top$
b) $\vec{x} \mapsto (2, 3)^\top$, $\vec{y} \mapsto (0, 1)^\top$, $\vec{z} \mapsto (2, 5)^\top$
c) $\vec{x} \mapsto (1, 1)^\top$, $\vec{y} \mapsto (1, 1)^\top$, $\vec{z} \mapsto (3, 3)^\top$

Problem # 45
a) Find the pattern behind the Fibonacci sequence: $1, 1, 2, 3, 5, 8, 13, \ldots$ and express it as a recursive formula.
b) Let f_i be the i-th term in the sequence. Let

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad \vec{x}_i = \begin{pmatrix} f_i \\ f_{i-1} \end{pmatrix} \quad \text{for } i \geq 2.$$

Show that $A\vec{x}_i = \vec{x}_{i+1}$.
c) Determine the eigenvalues λ_1, λ_2 and corresponding eigenvectors $\vec{a}, \vec{b}$ of the matrix A.
d) Put $S := (\vec{a} \mid \vec{b})$. Compute $S^{-1}AS$.
e) Find a formula for A^2. Find a formula for A^3. Find a formula for A^n.
f) Verify that $\vec{x}_{n+1} = A^n \vec{x}_1$.
g) Find a formula for f_n for general n.